1 Vetnisatóm er róteind + e punkthleðsla og hleðsludreifing rafeindar

$$\rho(r) = -\frac{e}{\pi a_0^3} \exp\left\{-\frac{2r}{a_0}\right\}$$

a) Heildarhleðsla H-atóms innan kúlu ~~með~~ geisla r ~~með~~ miðju í róteind.

$$Q(r) = e - \int \rho \, dV$$

heildunytin horuin θ og φ

Í kúluhnitum er $dV = 4\pi (r')^2 dr'$

$$Q(r) = e - e\frac{4\pi}{\pi a_0^3} \int_0^r (r')^2 dr' \, e^{-\frac{2r'}{a_0}}$$

$$= e\left\{1 - 4\int_0^{r/a_0} x^2 dx \, e^{-2x}\right\}$$

þar sem breytuskipti $x = r/a_0$ eru notuð

$$Q(r) = e\left\{1 - \left[(2x^2 + 2 + 1)e^{-2x}\right]\Big|_0^{r/a_0}\right\}$$

$$\boxed{Q(r) = e\left\{2\left(\frac{r}{a_0}\right)^2 + 2\left(\frac{r}{a_0}\right) + 1\right\} e^{-2\left(\frac{r}{a_0}\right)}}$$

b) Strax sést ~~að~~

$$\lim_{r \to \infty} Q(r) = 0$$

c) Finna $\bar{E}(r)$

Kúlusamhverf hleðsla → lögmál Gauss gefur

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q(r)}{\epsilon_0}$$

$Q(r)$ er jákvæð stærð → $\bar{E}$ stefnir út frá miðju atóms $\boxed{\bar{E} = E\hat{r}}$

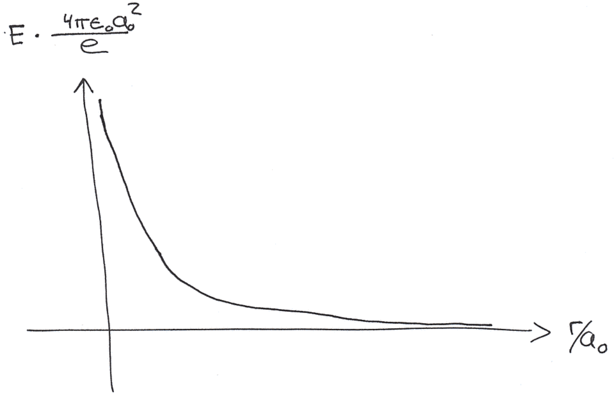
og $\bar{E} \cdot d\bar{A} = E\,dA$ á Gauss kúluyfirb.

$$E(r)(4\pi r^2) = \frac{Q(r)}{\epsilon_0}$$

$$\rightarrow \boxed{\bar{E}(r) = \hat{r}\left\{\frac{e}{4\pi\epsilon_0 r^2}\left[2\left(\frac{r}{a_0}\right)^2 + 2\left(\frac{r}{a_0}\right) + 1\right]e^{-2\left(\frac{r}{a_0}\right)}\right\}}$$

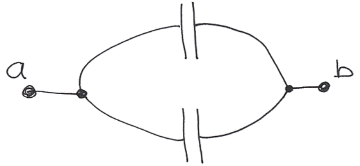
d)

$$E(r) = \frac{e}{4\pi\epsilon_0 a_0^2}\left[2 + \frac{2}{\left(\frac{r}{a_0}\right)} + \frac{1}{\left(\frac{r}{a_0}\right)^2}\right]e^{-2\left(\frac{r}{a_0}\right)}$$



2

$C_1 = 35\,uF$



$C_2 = 75\,uF$

Gefin V_{ab}

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a) Finna heildar Q_T

Tveir samsíða þéttar

$$C_T = C_1 + C_2 = 110\,uF$$

Almennt gildir $Q = CV$, $U = \frac{1}{2}CV^2$

$$Q_T = C_T V_{ab} = (C_1 + C_2) V_{ab}$$

---

b) finna Q_1 og Q_2

Hliðtengdir → sama spennufall V_{ab} yfir báða þétta

$$\rightarrow \quad Q_1 = C_1 V_{ab}$$

$$Q_2 = C_2 V_{ab}$$

c) Heildarorkan

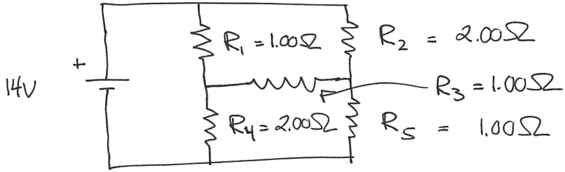
$$U_T = \frac{1}{2} C_T V_{ab}^2 = \frac{1}{2}(C_1 + C_2) V_{ab}^2$$

d) Orkan í hvorum þétti

$$U_1 = \frac{1}{2} C_1 V_{ab}^2 , \quad U_2 = \frac{1}{2} C_2 V_{ab}^2$$

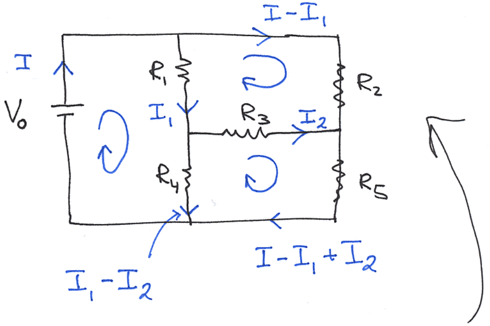
e) Spennan yfir hvorn þétti
þegar notað $V_1 = V_2 = V_{ab}$
því þeir eru hliðtengdir

3



finna straum i gegnum rathlöda og hvert ~~v~~ ~~o~~ ~~u~~ ~~v~~ ~~u~~ ~~a~~ ~~m~~

3 lykkjur



Nefnum straumna eins og hér sést og notum reglu Kirchhoffs fyrir spennufall í lykkjum

Vinstri lykkja

$$V_0 - R_1 I_1 - R_4(I_1 - I_2) = 0$$

$$\rightarrow \quad 14 - I_1 - 2(I_1 - I_2) = 0$$

$$\rightarrow \quad \boxed{3I_1 - 2I_2 = 14} \quad \text{I}$$

Topplykkja

$$-R_2(I - I_1) - (-I_2)R_3 - (-I_1)R_1 = 0$$

$$\rightarrow \quad -R_2(I - I_1) + I_2 R_3 + I_1 R_1 = 0$$

$$\rightarrow \quad -2(I - I_1) + I_2 + I_1 = 0$$

$$\rightarrow \quad \boxed{-2I + 3I_1 + I_2 = 0} \quad \text{II}$$

Botnlykkja

$$-R_5(I - I_1 + I_2) - R_4(I_2 - I_1) - R_3 I_2 = 0$$

$$\rightarrow \quad -(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$$

$$\rightarrow \quad \boxed{-I + 3I_1 - 4I_2 = 0} \quad \text{III}$$

leysum saman I II og III

$I = 10\ A$ ($\leftarrow$ í gegnum rafhlöðu)

$I_1 = 6\ A$ (í gegnum R_1)

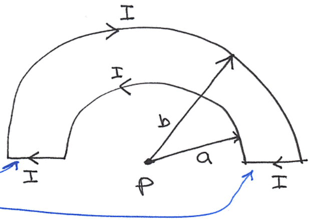
$I_2 = 2A$ (í -"- R_3)

$I_{R_2} = 4\ A$, $I_{R_4} = 4A$, $I_{R_5} = 6A$

Hvert er jafngildis ~~v~~viðnám flétunnar

$$R_{eq} = \frac{V_0}{I} = \frac{14\,V}{10\,A} = 1{,}40\,\Omega$$

4



$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{I d\bar{l} \times \hat{r}}{r^2}$$

fyrir beinu kaflarna er $d\bar{l} \times \hat{r} = 0$, tí $d\bar{l}$ og $\hat{r}$ eru samsíða fyri P

Í bók er leitt út segulsvið inni í miðjum hring við geisla R

$$B = \frac{\mu_0 I}{2R}$$

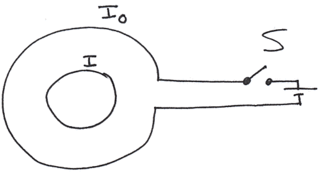
Samsvarandi gevur tað fyri P í hálvhring: fast

$$B = \frac{\mu_0 I}{4R}$$

Her eru streymstefnur mismunandi í hh

$$\rightarrow \boxed{B = B_a - B_b = \frac{\mu_0 I}{4}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ út úr blaðsíðu}}$$

5



a) Stefna I rétt eftir lokun S

I_0 ↺ → B út úr blaðinu

er vaxandi I reynir að vinna á móti breytingunni → I ↻
(réttsælis)

b) I eftir að S er lengi lokaður

Engin breyting á I og segulflæði

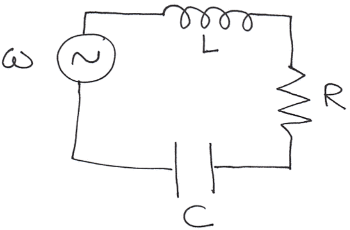
$\frac{d\Phi}{dt} = 0$ → $I = 0$

c) I rétt eftir að S er opnaður aftur

Nú vill I viðhalda segulsviðinu

→ I ↺ (rangsælis)

6 L-R-C-rás



$$X = X_L - X_C$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

a) Sýna að $X = 0$ þ. $\omega = \omega_0$

Hermitíðni $\omega_0 = \frac{1}{\sqrt{LC}}$

Ef $\omega = \omega_0$ þá fæst

$$X = \omega_0 L - \frac{1}{\omega_0 C} =$$

$$= \frac{L}{\sqrt{LC}} - \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0$$

---

b) Formerki X þ. $\omega > \omega_0$

$$X = \omega L - \frac{1}{\omega C} = \left(\frac{\omega}{\omega_0}\right) L\omega_0 - \left(\frac{\omega_0}{\omega}\right)\frac{1}{C\omega_0}$$

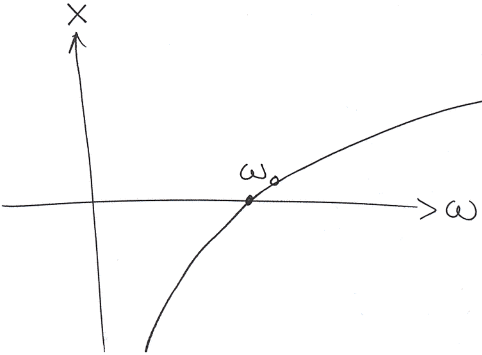
$$X = \left(\frac{\omega}{\omega_0}\right)\sqrt{\frac{L}{C}} - \left(\frac{\omega_0}{\omega}\right)\sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}}\left\{\left(\frac{\omega}{\omega_0}\right) - \left(\frac{\omega_0}{\omega}\right)\right\}$$

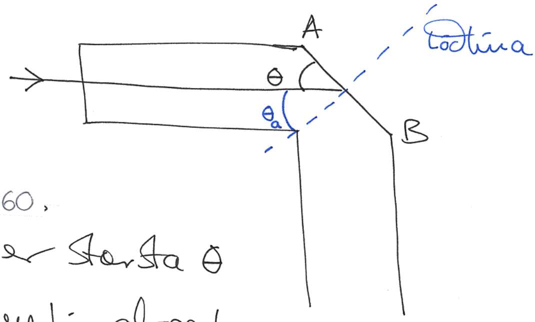
Ef $\omega > \omega_0 \longrightarrow X > 0$

c) Ef $\omega < \omega_0 \longrightarrow X < 0$

d)



(7)



a)

$n_a = 1.60$,

hvert er stærsta θ

sem leyfir alspeglun

Í reglu Snells er leitað að θ_a

sem gefur $\theta_b = \pi/2$

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$n_a = 1{,}60$, $\quad \theta_a + \theta = \frac{\pi}{2}$, $\quad n_b = 1$

$$n_a \sin\left(\frac{\pi}{2} - \theta\right) = 1$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{n_a}$$

$$\rightarrow \quad \cos\theta = \frac{1}{n_a} \quad \rightarrow \quad \theta = \arccos\left(\frac{1}{n_a}\right)$$

$$\theta = \arccos\left(\frac{1}{1{,}60}\right) \approx 0{,}9 \sim \underline{51^\circ}$$

b) Ef rörið er í vatni

$n_b = 1{,}33$

$$n_a \sin\left(\frac{\pi}{2} - \theta\right) = n_b \cdot 1$$

$$\rightarrow \sin\left(\frac{\pi}{2} - \theta\right) = \frac{n_b}{n_a}$$

$$\cos\theta = \frac{n_b}{n_a}$$

$$\theta = \arccos\left(\frac{n_b}{n_a}\right) = \arccos\left(\frac{1{,}33}{1{,}60}\right)$$

$$= 0{,}59 \sim \underline{\underline{34^\circ}}$$