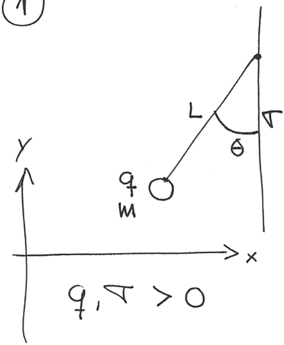
1



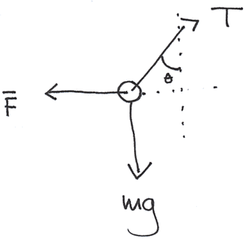
einangrandi flötur

Kúlan er í rafsviði flatarins

$$\bar{E} = -\hat{x}\,\frac{\sigma}{2\epsilon_0}$$

sem leiða má út frá lögmáli Gauss

Kraftar sem verka á kúlu eru



Við eigum að finna θ

$$\bar{F} = q\bar{E}$$

Greinum kraftana í þætti

x-þáttur :

$$F_x + T_x = 0$$

$$-q\frac{\sigma}{2\epsilon_0} + T\sin\theta = 0$$

y-þáttur :

$$-mg + T\cos\theta = 0$$

notum þessar tvær jöfnur til að fá

$$\tan\theta = \frac{T\cdot\sin\theta}{T\cdot\cos\theta} = \frac{q\sigma}{2\epsilon_0 mg}$$

$$\rightarrow \theta = \arctan\left(\frac{q\sigma}{2\epsilon_0 mg}\right)$$

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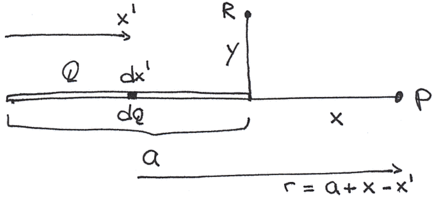
(2)



a) finna V í P

Skipta stönginni upp í lengdarfrymi dx' hvert með hleðslu $dQ = Q\frac{dx'}{a}$

fjarlægðin frá dx' til P er

$$r = a + x - x'$$

Mættið frá dQ er

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{dx'}{a+x-x'}$$

Heildum yfir öll dQ' (sem sé dx' frá $x'=0$ upp í $x'=a$)

$$V = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{a+x-x'}$$

$$= \frac{Q}{4\pi\epsilon_0 a}\left(-\ln(a+x-x')\Big|_0^a\right)$$

$$= \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right)$$

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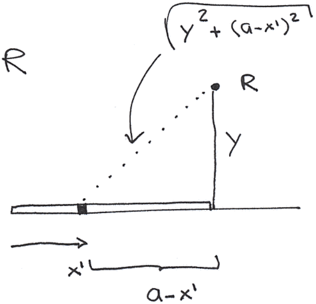
b) Finna V í R



$$dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \frac{dx'}{\sqrt{y^2 - (a-x')^2}}$$

$$V = \frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{dx'}{\sqrt{y^2 + (a-x')^2}}$$

notum breytuskipti $u = a - x'$, $du = -dx'$

$$\rightarrow \quad V = -\frac{Q}{4\pi\epsilon_0 a} \int_a^0 \frac{du}{\sqrt{y^2 + u^2}}$$

$$= -\frac{Q}{4\pi\epsilon_0 a} \left\{ \ln\left(u + \sqrt{y^2 + u^2}\right) \Big|_a^0 \right\}$$

$$= -\frac{Q}{4\pi\epsilon_0 a} \left\{ \ln y - \ln\left(a + \sqrt{y^2 + a^2}\right) \right\}$$

$$= \frac{Q}{4\pi\epsilon_0 a} \ln\left\{ \frac{a + \sqrt{y^2 + a^2}}{y} \right\}$$

c)

$$V(P) = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right)$$

$$\xrightarrow[x \gg a]{} 0$$

$$V(R) = \frac{Q}{4\pi\epsilon_0 a} \ln\left\{\frac{a+\sqrt{y^2+a^2}}{y}\right\}$$

$$\xrightarrow[y \gg a]{} 0$$

3

Tveir eins þéttar C

Ein spennugjafi V

a) Bera saman U_{Total} fyrir hlið- og röð tengingu

$$U_T = \frac{1}{2} C_T V^2$$

<ins>Raðtenging</ins>

$$C_T^{-1} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$\rightarrow C_T = \frac{C}{2}$$

Heildar spennufallið yfir C_T er V

$$\rightarrow U_T^R = \frac{1}{2} C_T V^2 = \frac{1}{4} C V^2$$

<ins>Hliðtenging</ins>

$$C_T = C + C = 2C$$

$$\rightarrow U_T^H = \frac{1}{2} C_T V^2 = C V^2$$

b) Bera saman heildar hleðsluna Q_T

$$Q_T = C_T V$$

Röð:

$$Q_T^R = \frac{C}{2} V = \frac{1}{2} CV$$

Hlið:

$$Q_T^H = 2CV = 2CV$$

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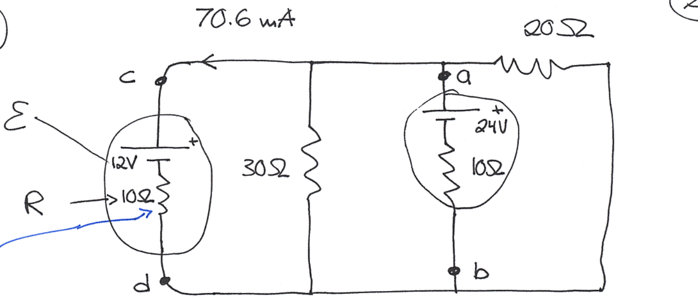
c) Ef þetta eru plötuþéttar gildir $E = V/d$ ← bil platna

$$E_R = V/(2d)$$

$$E_H = V/d$$

$$\longrightarrow \quad E_H = 2E_R$$

4



finna pólspennu 24-V rafhlöðunnar V_{ab}

Punktar a og c eru við sömu spennu

— || — d og b — || —

$\hookrightarrow V_{ab} = V_{cd}$

Straumurinn I hér er gefinn

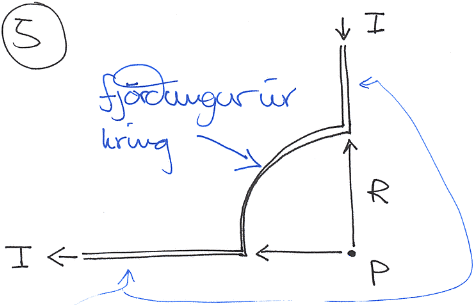
$I = 0{,}0706\ A$

$$V_d + IR + \mathcal{E} = V_c$$

$$\rightarrow V_c - V_d = V_{cd} = IR + \mathcal{E} = 0{,}0706\,A \cdot 10\,\Omega + 12\,V \approx 12.7\,V$$

→ $V_{ab} = V_a - V_b \approx 12.7\,V$

5



finna segulsviðið í P

Segulsviðið vegna straumfrymis $I d\bar{l}$ er

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{I d\bar{l} \times \hat{r}}{r^2}$$

þarsem r er fjarlægðin frá straumfryminu til athuganda (hér P) $\hat{r}$ er stefnan til P.

→ fyrir beinu leiðarana gildir

$d\bar{l}$ og $\hat{r}$ eru samsíða

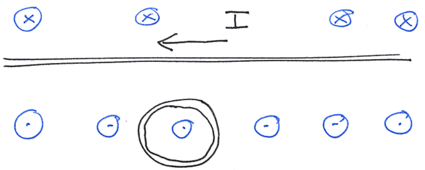
→ $d\bar{l} \times \hat{r} = 0$

þá er aðeins eftir fjórðungs-hringurinn

$$\rightarrow B(P) = \frac{1}{4}\left\{\frac{\mu_0 I}{2R}\right\} = \frac{\mu_0 I}{8R}$$

og í stefnu út úr blaðinu.

6



$$I(t) = I_0 e^{-bt}, \quad b>0$$

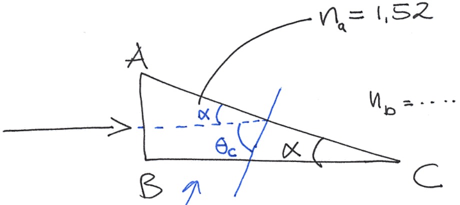
Finna stefnu I_{ind} í hringnum, $t>0$

Segulsviðið er upp í gegnum hringinn og minnkandi

$\hookrightarrow$ I_{ind} ↺ andsólis

til þess að viðhalda segulflæðinu um hringinn

7



Finna stærsta α sem leyfi að geislinn fari ekki út um AC

$$\to \quad \alpha + \theta_c = \frac{\pi}{2}$$

$$n_a \sin\theta_a = n_b \sin\theta_b$$

alspeglun $\to$ $\theta_b = \frac{\pi}{2}$

$$n_a \sin\theta_c = n_b$$

$$\sin\theta_c = \frac{n_b}{n_a}$$

$$\theta_c = \arcsin\left(\frac{n_b}{n_a}\right)$$

Der $\theta_c = \frac{\pi}{2} - \alpha$

$$\rightarrow \quad \frac{\pi}{2} - \alpha_{max} = \arcsin\left(\frac{n_b}{n_a}\right)$$

$$\alpha_{max} = \frac{\pi}{2} - \arcsin\left(\frac{n_b}{n_a}\right)$$

a) Luft $\rightarrow$ $n_a = 1{,}52$

$n_b = 1{,}00$

$$\rightarrow \alpha_{max} = \frac{\pi}{2} - 0{,}718 \simeq 0{,}853$$

$$\simeq 48{,}8°$$

b) i vatn $\rightarrow$ $n_a = 1{,}52$

$n_b = 1{,}33$

$$\rightarrow \alpha_{max} = \frac{\pi}{2} - 1.065 \simeq 0{,}506$$

$$\simeq 28{,}9°$$