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Forsendur Einsteins:

1. "Öll náttúrufyrirbæri hlíta sömu almennu lögmálum í öllum tregðukerfum
2. Ljóshraðinn er óháður hraða uppsprettunnar.

Hvaða varpanir samrýmast forsendunum?

$$(t, x, y, z) \to (t', x', y', z')$$

Athugum ljósblossa (frá S) á stað (x^*, y^*, z^*) klukkan $t = t^*$

Á t er bylgjuflöturinn kúla með geisla $c(t - t^*)$

2

þ.e.

$$c^2(t-t^*)^2 = (x-x^*)^2 + (y-y^*)^2 + (z-z^*)^2$$

forsendur $\downarrow$

$$c^2(t'-t^{*'}) = (x'-x^{*'})^2 + (y-y^{*'})^2 + (z-z^{*'})^2$$

Hvaða varpanir uppfylla þetta skilyrði?

Oft er gert ráð fyrir línulegri vörpun

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t + \delta(x))$$

hér skulum við athuga hvernig þetta má leiða út (hjálpa— með lausn)

Þá fæst:

$$\boxed{\begin{aligned} x' &= \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}(x - vt) \\ y' &= y, \quad z' = z \\ t' &= \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\left(t - \frac{v}{c^2}x\right) \end{aligned}}$$

Lorentz ummyndanir

Þægilegri ritháttur

$$x_0 \equiv ct$$
$$x_1 \equiv x$$
$$x_2 \equiv y$$
$$x_3 \equiv z$$

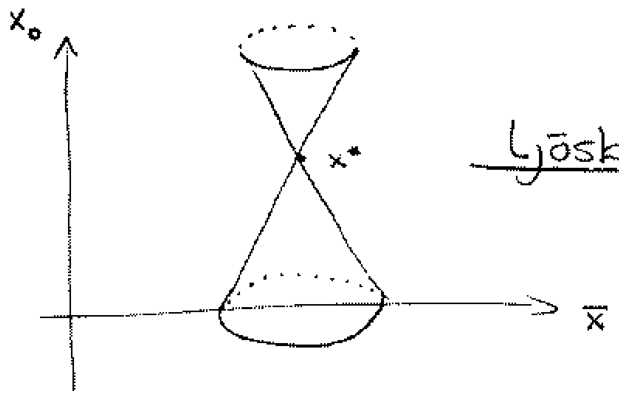
ekki hnitið x

og $\bar{x} = (x_1, x_2, x_3)$, $x = (x_0, x_1, x_2, x_3) \in \mathbb{R}^4$

Mengið

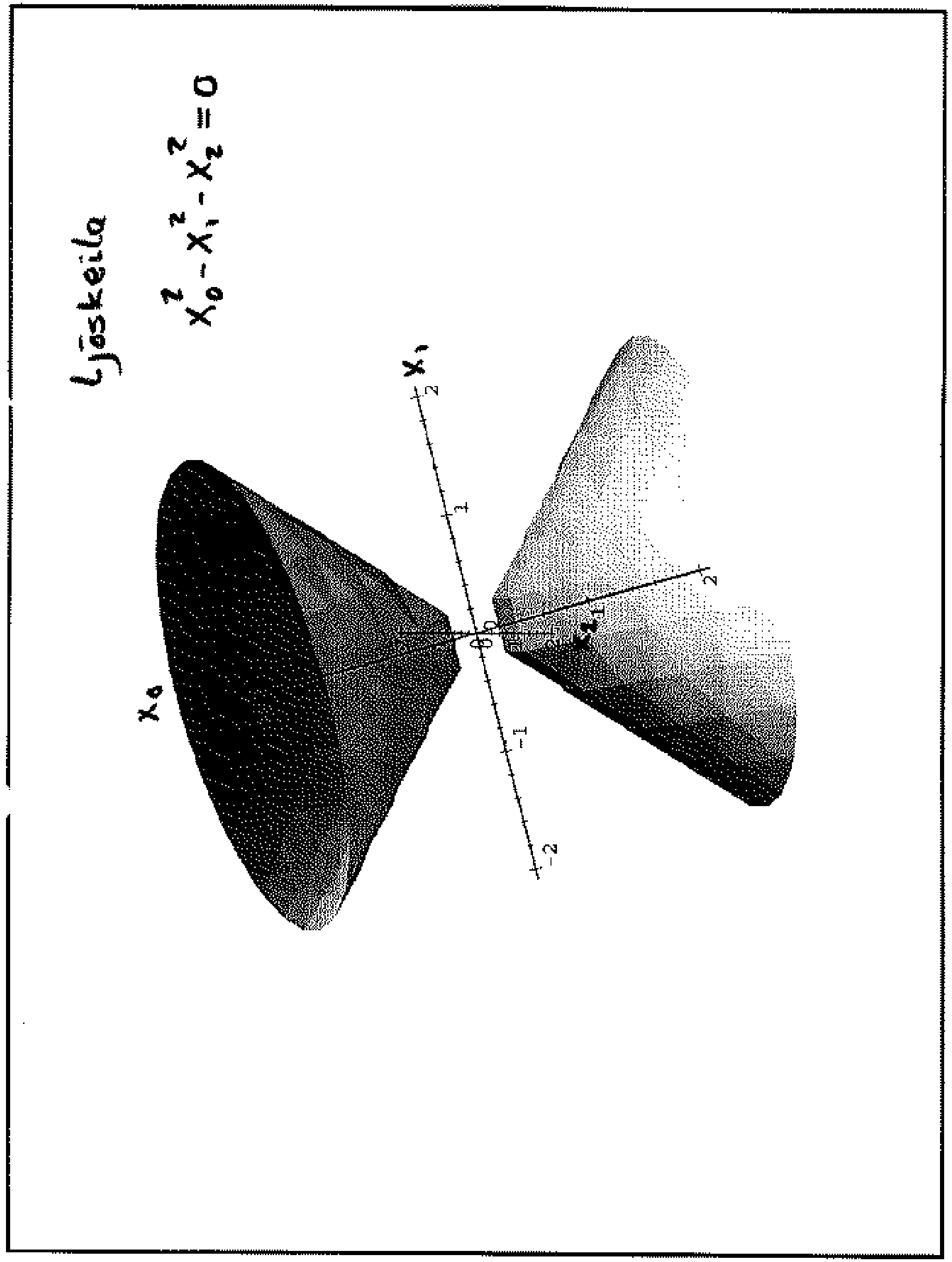
$$\left\{ x \in \mathbb{R}^4 \;\middle|\; (x_0 - x_0^*)^2 = (x_1 - x_1^*)^2 + (x_2 - x_2^*)^2 + (x_3 - x_3^*)^2 \right\}$$

$x^* = (x_0^*, \bar{x}^*) \in \mathbb{R}^4$ fastur p.



Ljóskeila

teiknuð í (1+2) víddum



Skilyrðið

$$0 = (x_0 - x_0^*)^2 - (x_1 - x_1^*)^2 - (x_2 - x_2^*)^2 - (x_3 - x_3^*)^2$$

jafngildir (forsendur Einsteins)

$$0 = (x_0' - x_0'^*)^2 - (x_1' - x_1'^*)^2 - (x_2' - x_2'^*)^2 - (x_3' - x_3'^*)^2$$

$$\Downarrow$$

Vörpunin sendir ljóskeilu í ljóskeilu

eða ljósgeisla $\longrightarrow$ ljósgeisla

Stærðfræði

slíkar varpanir má ávallt rita sem:

$$x' = Lx + a$$

$a \in \mathbb{R}^4$ fasti og L: línuleg vörpun $\mathbb{R}^4 \to \mathbb{R}^4$

Sem uppfyllir

$$\boxed{\begin{array}{l} (Lx)_0^2 - (Lx)_1^2 - (Lx)_2^2 - (Lx)_3^2 = 0 \\ \forall x \in \mathbb{R}^4 \iff x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 \end{array}}$$

$$\Big\downarrow \text{ Jafngilt}$$

$$(Lx)_0^2 - (Lx)_1^2 - (Lx)_2^2 - (Lx)_3^2 = \lambda(x_0^2 - x_1^2 - x_2^2 - x_3^2)$$

og $\lambda \neq 0$

nú þarf að sýna að $\lambda = 1$

L lýsir $S \longrightarrow S'$

S' hreyfist með hraða v miðað við S

afstæðislögmálið $\longrightarrow \lambda = \lambda(v)$

einungis

(6)

þ.a.

$$x_0'^2 - x_1'^2 - x_2'^2 - x_3'^2 = \lambda(v)(x_0^2 - x_1^2 - x_2^2 - x_3^2)$$

Vörpum $S' \to S$ þ.e. L^{-1} gefi

$$(x_0^2 - x_1^2 - x_2^2 - x_3^2) = \lambda(-v)(x_0'^2 - x_1'^2 - x_2'^2 - x_3'^2)$$

$$\to \quad \lambda(v) \cdot \lambda(-v) = 1 \quad (\text{gagntæk!})$$

Snúum S um hnitaás hornrétt á hreyfinguna

$$\to \quad v \to -v$$

$$\to \quad \lambda(-v) = \lambda(v) \quad \text{og} \quad \lambda^2(v) = 1$$

$$\hookrightarrow \lambda = \pm 1$$

$\lambda(v)$ samfellt af v

og $\lambda(0) = 1$ $\Big\} \to \lambda = 1$

(7)

<u>Breytum rithætti</u> $L \to \Lambda$, þá

$$(\Lambda x)_0^2 - (\Lambda x)_1^2 - (\Lambda x)_2^2 - (\Lambda x)_3^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$$

Athugum hnitaskipti þ.a.

$x' = \Lambda x$ og $\left| \begin{array}{l} x_2' = x_2 \\ x_3' = x_3 \end{array} \right.$ (x-ásar hreyfist saman)

þá er

$$\Lambda = \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad a,b,c,d \in \mathbb{R}$$

og

$$\begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

og skilyrðið

$$x_0'^2 - x_1'^2 = x_0^2 - x_1^2$$

hvað gerist fyrir $\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\longrightarrow \begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}, \begin{pmatrix} b \\ d \end{pmatrix}, \begin{pmatrix} a+b \\ c+d \end{pmatrix}$$

notum $x_0'^2 - x_1'^2 = x_0^2 - x_1^2$

$$\longrightarrow \underbrace{\begin{matrix} a^2 - c^2 = 1 \\ b^2 - d^2 = -1 \\ ab - cd = 0 \end{matrix}}$$

$$\downarrow$$

$$a^2 = d^2 \;;\quad b^2 = c^2$$

$$a^2 - c^2 = 1$$

a, b, c, d eru samfelld föll af v með $b, c = 0$ ef $v = 0$ og engar speglanir

$$\downarrow$$

$$a = d \qquad b = c$$

$$a^2 - b^2 = 1$$

Skilgreinum þá α þ.a.

$$a = \cosh\alpha \qquad \alpha \in \mathbb{R}$$
$$b = -\sinh\alpha$$

því fæst:

$$\Lambda = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

<u>hvernig tengist α v?</u>

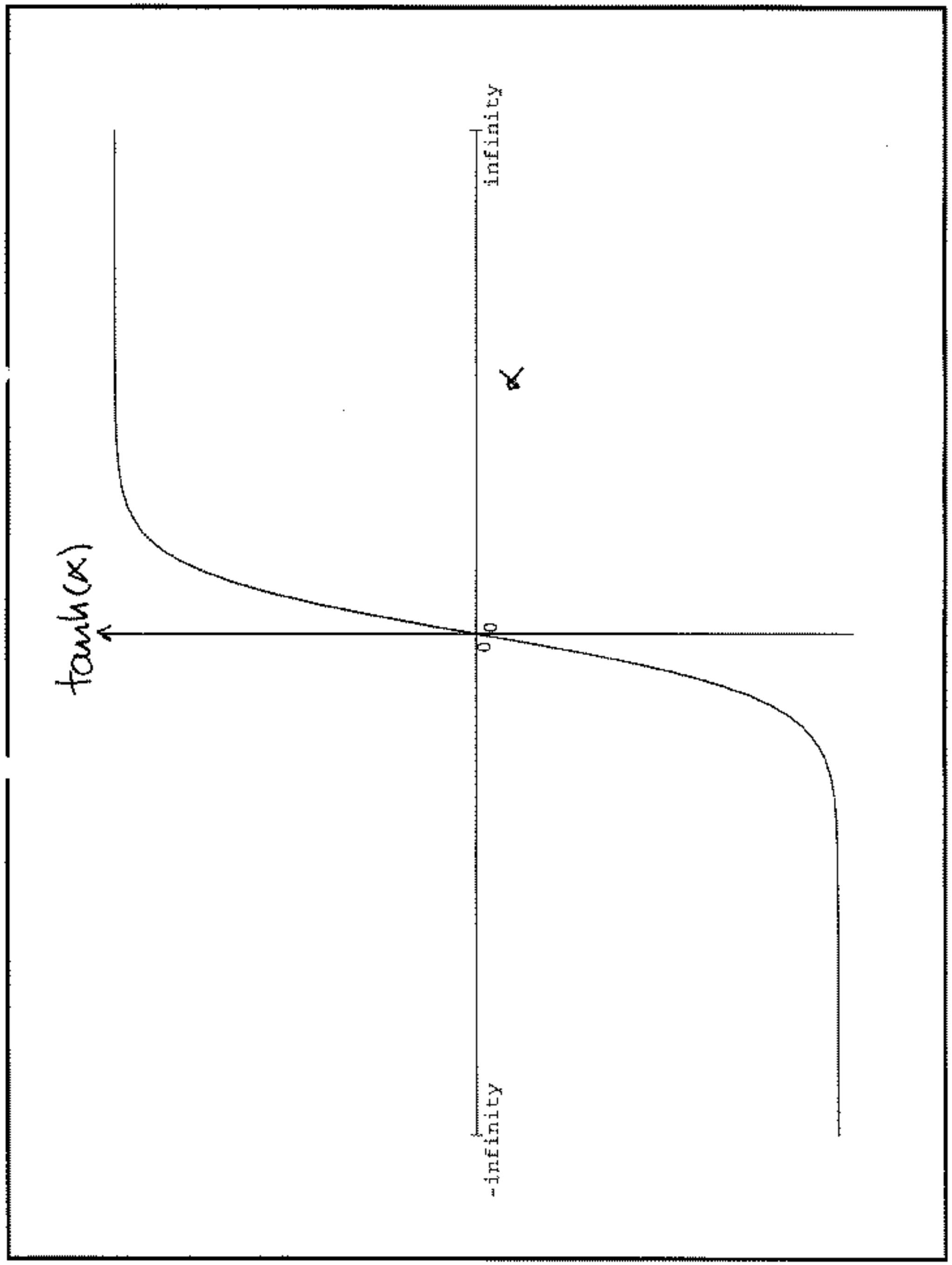
hvernig hreyfist $O_{S'}$ í S

í S' markar $\bar{O}_{S'}$ ferilinn $(x_0', \bar{0}) = x_0'$

í S er samsvarandi ferill $(x_0, \underbrace{v\frac{x_0}{c}}_{vt}, 0, 0)$

$$\| \\ x_0$$

Job: m3k913.a13018



$$\longrightarrow \begin{pmatrix} x_0' \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh\alpha & -\sinh\alpha \\ -\sinh\alpha & \cosh\alpha \end{pmatrix} \begin{pmatrix} x_0 \\ \frac{v}{c} x_0 \end{pmatrix}$$

ath

$$\longrightarrow -\sinh\alpha + \frac{v}{c}\cosh\alpha = 0$$

$$\downarrow$$

$$\frac{v}{c} = \tanh\alpha$$

α: ~~hraðastuðull~~ (rapidity)

$$\tanh\alpha < 1 \longleftrightarrow \underline{v < c}$$

notum

$$\cosh\alpha = \frac{1}{\sqrt{1-\tanh^2\alpha}}$$

$$\sinh\alpha = \frac{\tanh\alpha}{\sqrt{1-\tanh^2\alpha}}$$

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$$\Lambda = \begin{pmatrix} \frac{1}{\sqrt{1-v^2/c^2}} & \frac{-v/c}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ \frac{-v/c}{\sqrt{1-v^2/c^2}} & \frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$