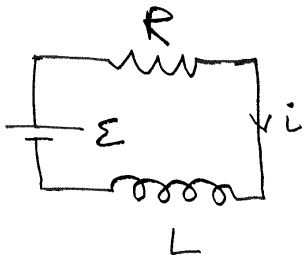
30 Orka í spólu, segulsvið

Kveikt á rás með spólu



Kirchhoff

$$\rightarrow \quad \varepsilon = iR + L\frac{di}{dt}$$

margfaldum með i

$$\rightarrow \quad i\varepsilon = i^2R + Li\frac{di}{dt}$$

- $i\varepsilon$: afl rafhlöðu
- i^2R: afl eytt í viðnámi
- $Li\frac{di}{dt}$: afl spólu ↕ orka/tímaeining komið fyrir í spólu

$$\frac{dU_L}{dt} = Li\frac{di}{dt}$$

heildarorkan í spólu þegar i hefur aukist frá 0 up í I

$$dU_L = Li\,di$$

$$\rightarrow \quad U_L = \int_0^{U_L} dU_L' = L\int_0^I i\,di$$

$$= \frac{1}{2}LI^2$$

Orka geymd í spólu

borið saman við orku í þétti

$$U_C = \frac{1}{2}\left(\frac{1}{C}\right)Q^2$$

{ eða hreyfiorku agna $E_{kin} = \frac{1}{2}mv^2$ }

Athugum sértilfelli

Löng spóla $\rightarrow L = \mu_0 n^2 A l$

$$n = \frac{N}{l}$$

segulsvið inni í spólu $B = \mu_0 n I$

$$\rightarrow U_L = \frac{1}{2}\underbrace{(\mu_0 n^2 A l)}_{L} I^2$$

$$= \frac{(\mu_0 n I)^2}{2\mu_0} \underbrace{Al}_{\text{rúmmál spólu}}$$

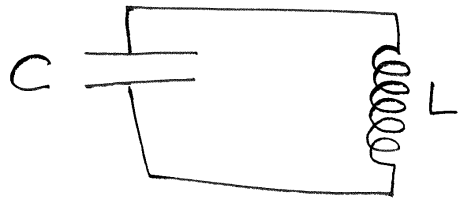
$$= \frac{B^2}{2\mu_0} Al$$

$\rightarrow$ orkuþéttleiki $u = \frac{B^2}{2\mu_0}$

almennt gildir um segulsvið

$$u_B = \frac{B^2}{2\mu_0}$$

LC - sveifla



- Ef í upphafi C er fullhlaðin (Q_0) $\rightarrow I = 0$
- síðan flæðir straumur um rásina C afhleðst
- Hámarks straumur (I_0) þegar $Q = 0$
- straumur hleður aftur C

$\rightarrow$ orka öll í L
$\rightarrow$ orka öll í C

} Orka sveiflast milli Rafsviðs og Segulsviðs

Athugum núna

Kirchhoff

$$V_c + \Sigma_L = 0$$

(Σ_L: sjálfspan)

$$\downarrow$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

Straumurinn er vegna breytinga á Q

$$I = -\frac{dQ}{dt}$$

I eykst þegar Q minnkar

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

diffurjafna fyrir Q

$$\boxed{\ddot{Q} + \frac{1}{LC}Q = 0}$$

ef $Q = Q_0$ fyrir $t = 0$

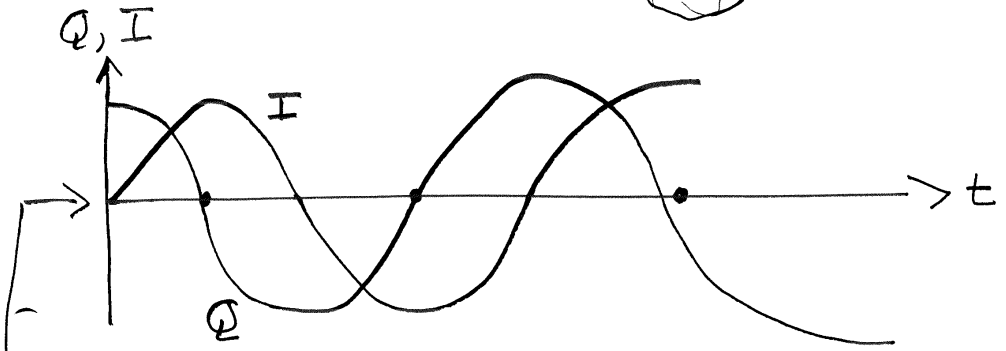
$Q(0) = Q_0$ ← max hleðsla

þá er lausnin

$$Q(t) = Q_0 \cos(\omega_0 t)$$

$$\rightarrow I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t)$$

$$= I_0 \sin(\omega_0 t)$$



max straumur $\boxed{I_0 = \omega_0 Q_0}$

náttúruleg tíðni $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

sést með umsetningu í deiftur jöfnuna

Hér sést að $I(t) = I_0$ þegar $Q(t) = 0$ og $Q(t) = Q_0$ þegar $I(t) = 0$

$$Q(t) = Q_0 \cos(\omega_0 t)$$

$$I(t) = I_0 \sin(\omega_0 t) = I_0 \cos(\omega_0 t - \tfrac{\pi}{2})$$

$I(t)$ og $Q(t)$ eru $\frac{\pi}{2}$ úr fasa

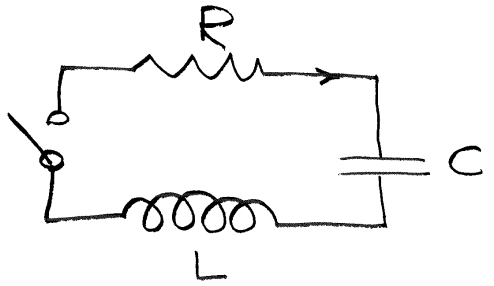
Í rauninni rás eydist orka
í viðnámi rásar og vegna

raftaugulgeislunar

↑

þegar hleðslum er hrædd

Deyfðar LC sveifla



$t=0$ rás lokuð

$Q(0) = Q_0$ hámarks hleðsla C

Kirchhoff

$$\rightarrow \frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

með $I = -\frac{dQ}{dt}$ (vaxandi I þegar Q minnkar)

$$\rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\rightarrow \boxed{\ddot{Q} + \frac{R}{L}\dot{Q} + \frac{1}{LC}Q = 0} \quad (*)$$

Ef lausnin hefur formið

$$\boxed{Q(t) = Q_0 e^{-\gamma t}\cos(\omega' t + \delta)}$$

þá gildir

$$\dot{Q} = -Q_0 e^{-\gamma t}\left\{\gamma\cos(\omega' t+\delta) + \omega'\sin(\omega' t+\delta)\right\}$$

$$\ddot{Q} = Q_0 e^{-\gamma t}\left\{(\gamma^2-(\omega')^2)\cos(\omega' t+\delta) + 2\gamma\omega'\sin(\omega' t+\delta)\right\}$$

til þess að (*) sé uppfyllt verður að gilda

$$(\ldots\ldots)\sin() + (\ldots\ldots)\cos() = 0$$

($=0$ og $=0$)

$$\boxed{-\omega'\frac{R}{L} + 2\gamma\omega' = 0 \rightarrow \gamma = \frac{R}{2L}}$$

$$\boxed{(\gamma^2-(\omega')^2) - \gamma\frac{R}{L} + \frac{1}{LC} = 0}$$

$$\hookrightarrow (\omega')^2 = \omega_0^2 - \gamma^2$$

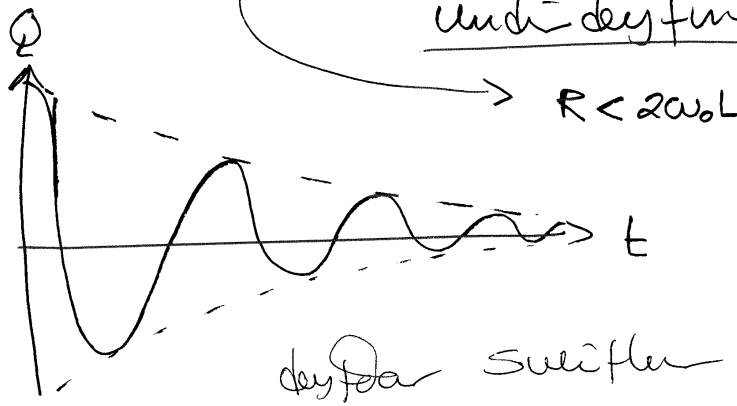
$$(\omega')^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2$$

↑ náttúrulega tíðnin ω_0

Ef $\omega_0 > \gamma \rightarrow \omega'$ er rauntala

undirdeyfing

$R < 2\omega_0 L$

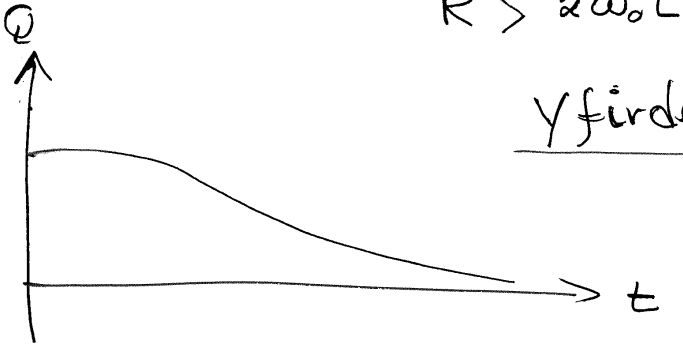


deyfðar sveiflur

Ef $\omega_0 < \gamma \rightarrow \omega'$ er tvíntala

$R > 2\omega_0 L$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

yfirdeyfing



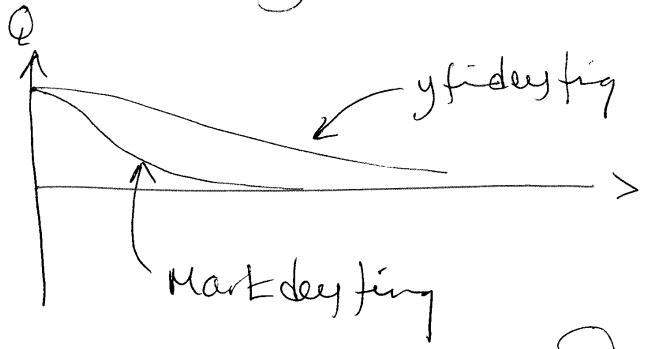
engar sveiflur, hog dofnun

Markgildi

$$\omega_0 = \gamma$$

$$L \Rightarrow \quad R = 2\omega_0 L$$

þarf að athuga sérstaklega



enga sveiflur, hröð dofnun

Nothum t.d. Hurða pumpa