Hallhrif

Klassískt Drude líkan fyrir rafeindir í efni

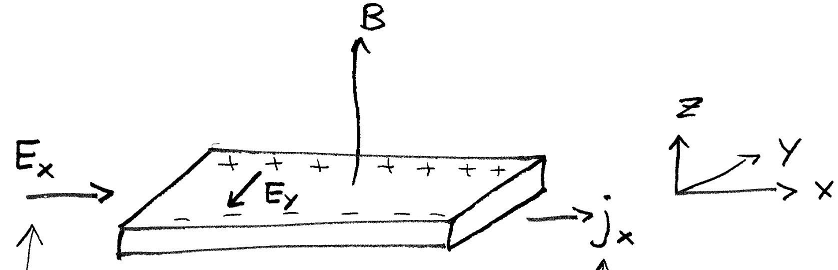
Fyrir eina eind gildir

$$\underbrace{\frac{d\bar{p}(t)}{dt}}_{\text{Skriðþungabreyting}} = \underbrace{\bar{F}(t)}_{\text{Kraftur vegna raf og segulsviðs.}} - \underbrace{\frac{1}{\tau}\bar{p}(t)}_{\text{núningur vegna árekstra við óhreinindi}}$$

Skriðþunga-breyting

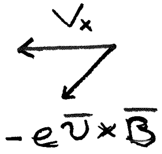
Kraftur vegna raf og segul-sviðs.

núningur vegna árekstra við óhreinindi
τ: slökunartími
(tími milli árekstra)



svið til að reka straum í gegnum efnið

hraði rafeinda $\longrightarrow$



Segulsviðið sveigir rafeindirnar til vinstri,

misdreifing hleðslu kemur upp rafsviði E_y

þegar jafn stöðugur straumur flæðir um efnið gildir $\frac{dp}{dt} = 0$

$$\longrightarrow \quad \bar{F}(t) - \frac{1}{\tau}\bar{P}(t) = 0$$

$$-e\left\{\bar{E} + \frac{\bar{p}}{m}\times\bar{B}\right\} - \frac{\bar{p}(t)}{\tau} = 0$$

$\frac{\bar{p}}{m} = \bar{v}$

Greinum í kartísk hnit

$$(1) \qquad 0 = -eE_x - p_y \underbrace{\frac{eB}{m}}_{\omega_c} - \frac{p_x}{\tau}$$

$$(2) \qquad 0 = -eE_y + p_x \frac{eB}{m} - \frac{p_y}{\tau}$$

- Áður höfum við skilgreint straum-þéttleika og Drude leiðni

$$\bar{J} = -n e \bar{v}_d = \sigma_0 \bar{E}, \quad \sigma_0 = \frac{ne^2\tau}{m}$$

rafeinda þéttleiki — rekhraði

- þegar ekkert ytra segulsvið var.

Núna segulsvið

margföldum ① og ② með $-\frac{ne\tau}{m}$

$$① \rightarrow \sigma_0 E_x = \omega_c \tau j_y + j_x$$

$$② \rightarrow \sigma_0 E_y = -\omega_c \tau j_x + j_y$$

þessar jöfnur má skrifa sem

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

eða

$$\boxed{\bar{E} = \overleftrightarrow{\rho}\, \bar{J}}$$

eðlisviðnámsfylki

- stökin utan hornalínu $\rightarrow 0$ þ. $B \rightarrow 0$ (og þess vegna $\omega_c \rightarrow 0$)

segulsviðið breytir skalar ρ yfir í fylki $\overleftrightarrow{\rho}$

Snúum sambandinu við

$$(\overleftrightarrow{\rho})^{-1}\bar{E} = \bar{J}$$

eða

$$\boxed{\bar{J} = \overleftrightarrow{\sigma}\,\bar{E}}$$

með $$\overleftrightarrow{\sigma} = (\overleftrightarrow{\rho})^{-1} = \frac{\sigma_0}{1+\omega_c^2\tau^2}\begin{pmatrix}1 & -\omega_c\tau\\ \omega_c\tau & 1\end{pmatrix}$$

eðlisleiðnifylki

$$= \begin{pmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{yx} & \sigma_{yy}\end{pmatrix}$$

Stökum utan hornalínu hverfa þ. $B \to 0$

Við sjáum því að almennt gildir

$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}} \qquad (\sigma_{xy} = -\sigma_{yx})$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$

$$\sigma_{xy} = -\frac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}$$

$$= \frac{-\sigma_0(1+\omega_c^2\tau^2)+\sigma_0}{\omega_c\tau\,(1+(\omega_c\tau)^2)}$$

$$= -\frac{\sigma_0}{\omega_c\tau} + \frac{\sigma_0}{\omega_c\tau}\,\frac{1}{1+(\omega_c\tau)^2}$$

$$= -\frac{ne}{B} + \frac{1}{\omega_c\tau}\,\sigma_{xx}$$

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og

$$\sigma_{xy} = -\frac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2} = -\omega_c\tau\,\sigma_{xx}$$

$\boxed{\text{tengsl } \sigma_{xy} \text{ og } \sigma_{xx}}$

Í tilraun gildir $j_y = 0$

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y$$

$$j_y = \sigma_{yy} E_y + \sigma_{yx} E_x = 0$$

$$\rightarrow \frac{E_x}{E_y} = -\frac{\sigma_{yy}}{\sigma_{yx}}$$

$$\rightarrow \frac{j_x}{E_y} = \sigma_{xx} \frac{E_x}{E_y} + \sigma_{xy}$$

$$= \frac{\sigma_{xx}^2}{\sigma_{xy}} + \sigma_{xy}$$

$$\left\{ = -\frac{\sigma_{xx}^2}{\omega_c \tau \sigma_{xx}} + \sigma_{xy} = -\frac{\sigma_{xx}}{\omega_c \tau} + \sigma_{xy} \right\}$$

$$= -\frac{ne}{B}$$

það sem er mált

$$E_y = \rho_{yx} j_x + \rho_{yy} j_y$$

$$\rightarrow \frac{j_x}{E_y} = \frac{1}{\rho_{yx}}$$

Líka er $\boxed{\text{mált}}$

$$\frac{j_x}{E_x} = \sigma_{xx} + \sigma_{xy} \frac{E_y}{E_x}$$

$$= \sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{xx}}$$

$$\left\{ \begin{aligned} &= \sigma_{xx} + \omega_c^2 \tau^2 \sigma_{xx} \\ &= \sigma_{xx} (1 + (\omega_c \tau)^2) \end{aligned} \right\} = \sigma_0$$

Skilgreindur er Hall stuðull

$$\boxed{R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}}$$

Bein máling á rafeinda þettleika og formerki hleðslubera

Skammta Hallhrif (Klaus v. Klitzing 1980)

við lágt T og hátt B

í hálfleiðurum

Verður $$\boxed{\frac{j_x}{E_y} = -\,i\,\frac{e^2}{h}}$$

með $i = 1,2,3,\ldots$, h = Plancksfasti

og $$\boxed{\frac{j_x}{E_x} = 0 \quad \text{samtímis}}$$

$$\rightarrow \boxed{\sigma_{xx} = 0 \quad \text{og} \quad \sigma_{yx} = i\,\frac{e^2}{h}}$$

leiðni aðeins háð náttúruföstum

e og h

Nóbelsverðlaun 1985

$$\sigma_{xx} = 0 \;\rightarrow\; \rho_{xx} = 0$$

$$\rightarrow \boxed{\rho_{xy} = \frac{h}{i e^2}}$$

Náttúrulegur skammtur eðlisviðnáms

Leiðni og eðlisviðnámið í x-áttina eru samtímis 0, $\sigma_{xx} = 0$, $\rho_{xx} = 0$

↑ aðeins hægt þar sem um fyltis-staði er að ræða

Jafnvel án segulsviðs er viðnám og leiðni oft fyltis-staði

Viðnámsstaðall

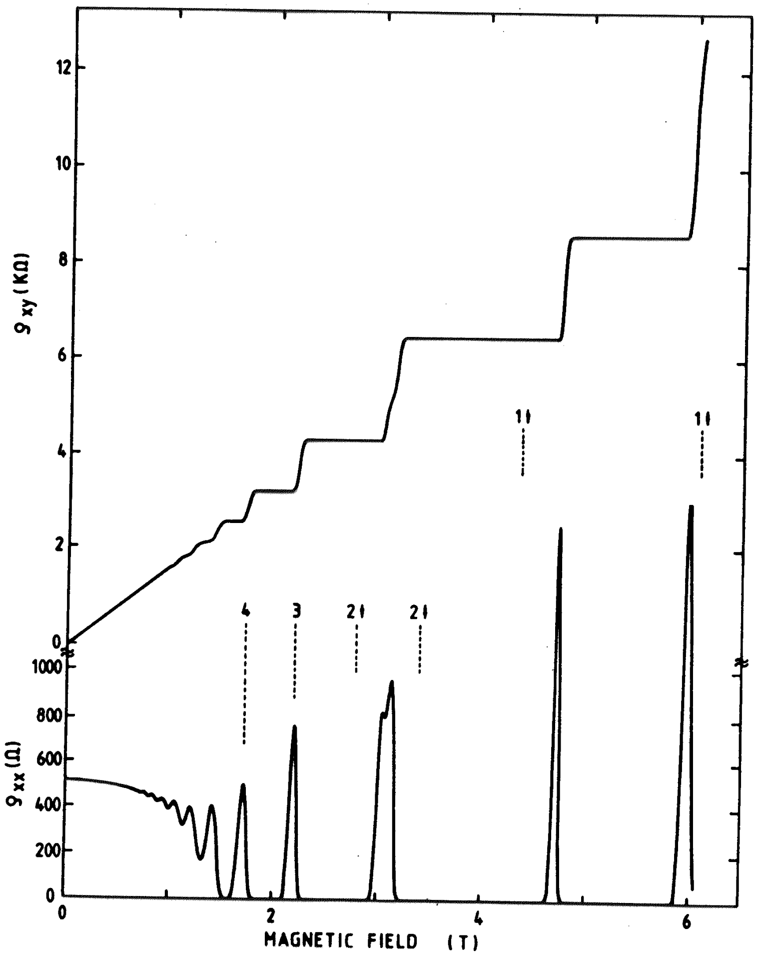


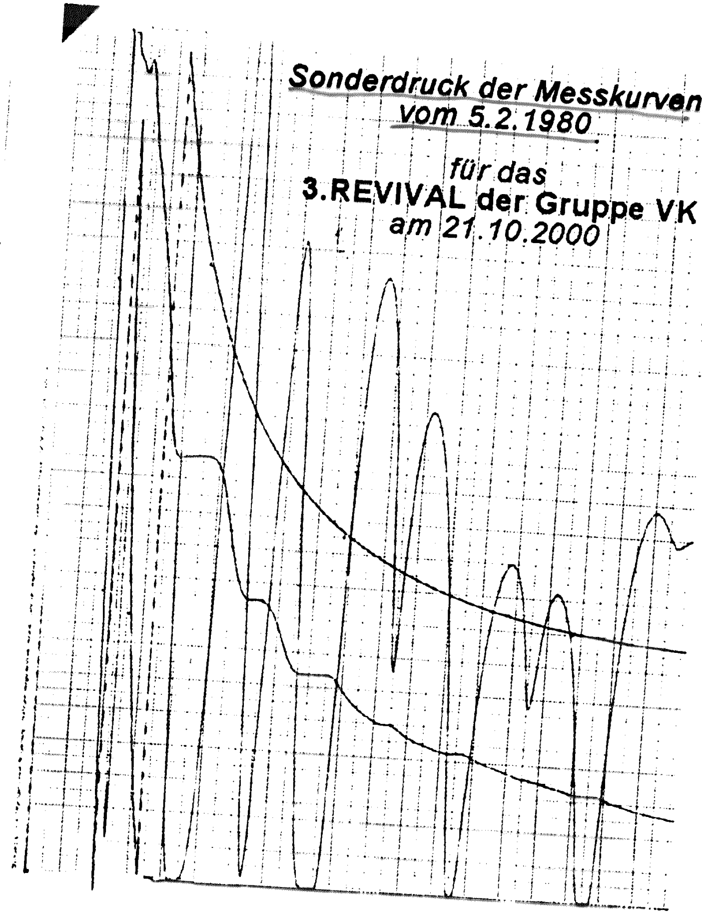
Fig. 14. Experimental curves for the Hall resistance $R_H = \varrho_{xy}$ and the resistivity $\varrho_{xx} \sim R_x$ of a heterostructure as a function of the magnetic field at a fixed carrier density corresponding to a gate voltage $V_g = 0V$. The temperature is about 8mK.

This analysis is based on the equation

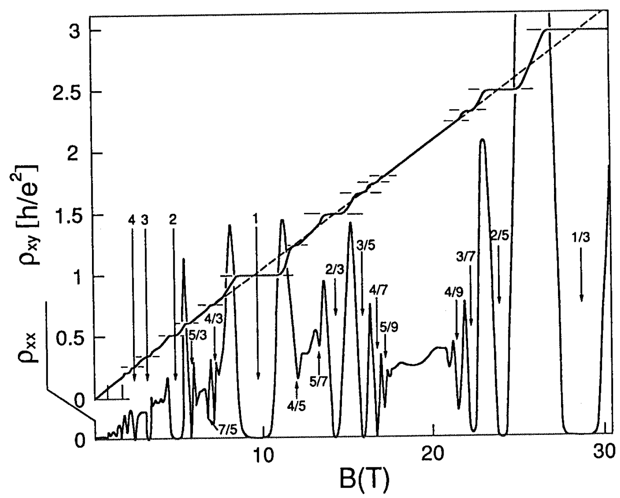
$$\frac{1}{C} = \frac{1}{e^2 \cdot D(E_F)} + \text{const.} \qquad (19)$$

The combination of the different methods for the determination of the DOS leads to a result as shown in Fig. (20). Similar results are obtained from other experiments, too [33, 34] but no theoretical explanation is available.

If one assumes that only the occupation of extended states influences the Hall effect, than the slope $d\varrho_{xy}/dn_s$ in the plateau region should be dominated



Ein, 1982 brottöluhrif



D.C. Tsui, H. Störmer, R.B. Laughlin
Nóbelsverðlaun 1998

$$\rho_{xy} = \frac{h}{i e^2}$$

$i = 1/3,\ 2/3,\ \ldots\ldots$

Heiltöluhrif: Landau-stig, staðbinding

Brottöluhrif: víxlverkun rafeinda
→ ósamþjappanlegur vökvi

Sýndareindir m. brothleðslu