③

$$\Phi(\bar{r}) = \int d\bar{r}'\, g(\bar{r}-\bar{r}')\, f(\bar{r}')$$

$$\hookrightarrow \quad \Phi(\bar{k}) = g(\bar{k})\, f(\bar{k})$$

$$\Phi(\bar{k}) = \int d\bar{r}'\, d\bar{r}\, e^{-i\bar{k}\cdot\bar{r}}\, g(\bar{r}-\bar{r}')\, f(\bar{r}')$$

$$= \frac{1}{(2\pi)^6} \int d\bar{r}'\, d\bar{r}\, d\bar{q}\, d\bar{l}\, e^{-i\bar{k}\cdot\bar{r}}\, g(\bar{q})\, f(\bar{l}) \cdot e^{i\bar{q}\cdot(\bar{r}-\bar{r}')}\, e^{i\bar{l}\cdot\bar{r}'}$$

$$= \frac{1}{(2\pi)^6} \int d\bar{r}'\, d\bar{r}\, d\bar{q}\, d\bar{l}\, e^{i\bar{r}\cdot(\bar{q}-\bar{k})}\, e^{i\bar{r}'(\bar{l}-\bar{q})}\, g(\bar{q})\, f(\bar{l})$$

$$= \int d\bar{q}\, d\bar{l}\, \delta(\bar{q}-\bar{k})\, \delta(\bar{l}-\bar{q})\, g(\bar{q})\, f(\bar{l})$$

$$= \int d\bar{q}\, \delta(\bar{q}-\bar{k})\, g(\bar{q})\, f(\bar{q}) = g(\bar{k})\, f(\bar{k})$$

(2) jöpuur (32) + (34)

$$\phi^{ext}(\bar{r}) = \frac{Q}{4\pi\epsilon_0 r}$$ ← kulusam-huerta

$$\phi^{ext}(\bar{k}) = \frac{Q}{\epsilon_0 k^2}$$

$$\phi^{ext}(\bar{k}) = \int d\bar{r}\, e^{-i\bar{k}\cdot\bar{r}} \frac{Q}{4\pi\epsilon_0 r}$$

$u = \cos\theta$

$$= \frac{2\pi Q}{4\pi\epsilon_0} \int_0^\infty r\,dr \int_{-1}^{1} du\; e^{-ikru}$$

$$= \frac{Q}{2\epsilon_0} \int_0^\infty r\,dr \left\{ \frac{e^{-ikru}}{-ikr} \Big|_{-1}^{1} \right\}$$

$$= \frac{Q}{\epsilon_0 k} \int_0^\infty dr\, \sin(kr)$$

bota vid sambitni stucli

$\mu \geq 0$

$$\phi^{ext}_{\mu}(k) = \frac{Q}{\epsilon_0 k} \int_0^\infty dr \, \mathrm{Sin}(kr) \, e^{-\mu r}$$

$\mu \to 0$

$$= \frac{Q}{\epsilon_0 k} \left\{ \frac{k}{k^2+\mu^2} \right\}$$

$$= \frac{Q}{\epsilon_0 k^2} = \frac{Q}{\epsilon_0 k^2}$$

$$\phi(\bar{r}) = \int \frac{d\bar{k}}{(2\pi)^3} e^{i\bar{k}\cdot\bar{r}} \frac{Q}{\epsilon_0 (k^2+k_0^2)}$$

$$= \frac{1}{(2\pi)^2} \int_0^\infty k^2 dk \int_{-1}^{1} du \, e^{ikru} \frac{Q}{\epsilon_0 (k^2+k_0^2)}$$

$$= \frac{Q}{4\pi^2 \epsilon_0} \int_0^\infty k^2 dk \int_{-1}^{1} du \, \frac{e^{ikru}}{k^2 + k_0^2}$$

$$= \frac{Q}{4\pi^2 \epsilon_0} \int_0^\infty \frac{k^2 dk}{k^2 + k_0^2} \left\{ \frac{e^{ikru}}{ikr} \Big|_{-1}^{1} \right\}$$

$$= \frac{2Q}{4\pi^2 \epsilon_0 r} \int_0^\infty \frac{k dk}{k^2 + k_0^2} \sin(kr)$$

$$= \frac{2Q}{4\pi^2 \epsilon_0 r} \frac{\pi}{2} e^{-k_0 r}$$

$$= \frac{Q e^{-k_0 r}}{4\pi \epsilon_0 r}$$

(3)

$$\epsilon_r(\omega) = 1 - \frac{\Omega^2}{\omega^2}$$

är med nollstöd

$$\omega^2 = \Omega^2$$
