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Kúla með hleðslu Q → $\rho(r) = \frac{3Q}{4\pi R^3}$ fasti

$$-\nabla^2 \phi(r) = \frac{\rho(r)}{\epsilon_0}$$

Kúlusamhverf einsleit hleðsludreifing

$$\rightarrow \quad -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\phi(r)\right) = \frac{\rho}{\epsilon_0}$$

Kúlan er ekki úr málmi, engin yfirborðshleðsla σ
(óendanlega þunnt lag)

→ ein breyta

$$\rightarrow \quad \frac{d}{dr}\left(r^2 \frac{d}{dr}\phi(r)\right) = -\frac{r^2 \rho}{\epsilon_0}$$

$$\bar{E} = -\bar{\nabla}\phi(r) = -\frac{\partial}{\partial r}\phi(r) \quad \text{hér}$$

Finnum lausn utan og innan kúlu

$\phi(r)$ er samfellt í $r = R$

Engin yfirborðshleðsla $\rightarrow$ $E_r(r)$ er líka samfellt í $r = R$

Í málmi má nota lögmál Gauß t.þ.a. finna að yfirborðshleðsla þar leiði til stökks í $E_r(r)$ í réttu hlutfalli við σ

Ein öäkveetin heitdan

$$r^2 \frac{d}{dr} \phi_o(r) = -\frac{r^3 \rho}{3\epsilon_0} + C_1$$ ytri lausn

$$r^2 \frac{d}{dr} \phi_i(r) = -\frac{r^3 \rho}{3\epsilon_0} + k_1$$ innri lausn

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$$\rightarrow \quad \frac{d}{dr} \phi_o(r) = -\frac{r \rho}{3\epsilon_0} + \frac{C_1}{r^2} \qquad r > R$$

$$\frac{d}{dr} \phi_i(r) = -\frac{r \rho}{3\epsilon_0} + \frac{k_1}{r^2} \qquad r < R$$

$\underline{r > R}$, $\rho = 0$ $\longrightarrow$ $$\frac{d}{dr}\phi_o(r) = \frac{C_1}{r^2}$$

$\underline{r < R}$, engin sérstöðu- punktur, engin punkt hleðsla $\Big\}$ $\longrightarrow$ $$\frac{d}{dr}\phi_i(r) = -\frac{r\rho}{3\epsilon_0}$$

$\subset$ þetta er r-þáttur rafsviðs sem er samfelldur í R

$$\frac{C_1}{R^2} = -\frac{R\rho}{3\epsilon_0} \longrightarrow C_1 = -\frac{R^3\rho}{3\epsilon_0}$$

$$= -\frac{R^3 3Q}{3\epsilon_0 4\pi R^3}$$

$$= -\frac{Q}{4\pi\epsilon_0}$$

Því höfum við

$\underline{r > R}$: $$\frac{d}{dr}\phi_o(r) = -\frac{Q}{4\pi\epsilon_o r^2}$$

$\underline{r < R}$: $$\frac{d}{dr}\phi_i(r) = -\frac{r\rho}{3\epsilon_o}$$

Enn ein óákveðin heildun

$$\phi_o(r) = \frac{Q}{4\pi\epsilon_o r} + C_2 \qquad \underline{r>R}$$

$$\phi_i(r) = -\frac{r^2\rho}{6\epsilon_o} + K_2 \qquad \underline{r<R}$$

Veljum 0-punkt fyrir $r \to \infty$ $\to$ $C_2 = 0$

Kretjunst sam feller i $r=R$

$$\frac{Q}{4\pi\epsilon_0 R} = -\frac{R^2\rho}{6\epsilon_0} + k_2$$

hēr er allt ~~pekkt~~ nema k_2

$$\to k_2 = \frac{Q}{4\pi\epsilon_0 R} + \frac{R^2\rho}{6\epsilon_0}$$

$$= \frac{Q}{4\pi\epsilon_0 R} + \frac{R^2\, 3Q}{6\epsilon_0\, 4\pi R^3}$$

$$= \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{R} + \frac{1}{2R}\right\} = \frac{Q}{4\pi\epsilon_0}\,\frac{3}{2R}$$

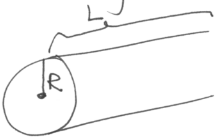
Lausnin innan kúlu er ϕ_i

$$\phi_i(r) = -\frac{r^2 \rho}{6\epsilon_0} + \frac{Q}{4\pi\epsilon_0}\frac{3}{2R}$$

$$= -\frac{r^2\, 3Q}{6\epsilon_0\, 4\pi R^3} + \frac{Q}{4\pi\epsilon_0}\frac{3}{2R}$$

$$= \frac{Q}{4\pi\epsilon_0\, 2R^3}\left\{3R^2 - r^2\right\}$$

Sívalningur



$$\rho = \frac{Q}{\pi R^2 L} = \frac{\lambda}{\pi R^2} \quad \text{þ.s. } \lambda = \frac{Q}{L}$$

Ekkert breytir hornsamhverfu

$$\rightarrow -\nabla^2 \phi(r) = \frac{\rho}{\epsilon_0} \quad \Leftrightarrow \quad -\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\phi\right) = -\frac{\rho}{\epsilon_0}$$

einbreyta

$$\rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\phi\right) = -\frac{\lambda}{\pi R^2 \epsilon_0}$$

$$\frac{d}{dr}\left(r\frac{d}{dr}\phi\right) = -\frac{\lambda r}{\pi R^2 \epsilon_0}$$

Óákveðin heildun

$$r\frac{d}{dr}\phi_o = -\frac{\lambda r^2}{2\pi R^2 \epsilon_0} + c_1 \qquad \underline{r > R}$$

$$r\frac{d}{dr}\phi_i = -\frac{\lambda r^2}{2\pi R^2 \epsilon_0} + k_1 \qquad \underline{r < R}$$

$$\frac{d}{dr}\phi_o = -\frac{\lambda r}{2\pi R^2 \epsilon_0} + \frac{c_1}{r} \qquad \underline{r > R}$$

$$\frac{d}{dr}\phi_i = -\frac{\lambda r}{2\pi R^2 \epsilon_0} + \frac{k_1}{r} \qquad \underline{r < R}$$

tengt rafsviðinu í $\hat{r}$ átt

Innan, engin sérstaða

$$\frac{d}{dr}\phi_i = -\frac{\lambda r}{2\pi R^2 \epsilon_0} \qquad r < R$$

utan, $\lambda = 0$

$$\frac{d}{dr}\phi_o = \frac{C_1}{r}$$

samfella í R

$$-\frac{\lambda R}{2\pi R^2 \epsilon_0} = \frac{C_1}{R} \longrightarrow C_1 = -\frac{\lambda}{2\pi \epsilon_0}$$

Við höfum því

$\underline{r < R}$:
$$\frac{d}{dr}\phi_i(r) = -\frac{\lambda r}{2\pi R^2 \epsilon_0}$$

$\underline{r > R}$:
$$\frac{d}{dr}\phi_o(r) = -\frac{\lambda}{2\pi \epsilon_0 r}$$

Óákveðin heildun

$\underline{r < R}$:
$$\phi_i(r) = -\frac{\lambda r^2}{4\pi R^2 \epsilon_0} + C_2$$

$\underline{r > R}$:
$$\phi_o(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln(r) + K_2$$

samfella í $r=R$ gefur annan fastann
Hinn fast frá ákvörðum um 0-punkt

ytri lausnin sýnir að ekki er hægt að setja hann í $R \to \infty$.

Hann má t.d. setja á yfirborði $r=R$ eða í $r=0$
veljum það síðara $\to C_2 = 0$ og samfellan verður

$$-\frac{\lambda R^2}{4\pi R^2 \epsilon_0} = -\frac{\lambda}{2\pi\epsilon_0}\ln R + K_2$$

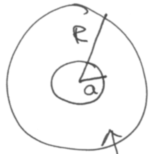
$$\to K_2 = \frac{\lambda}{2\pi\epsilon_0}\ln R - \frac{\lambda R^2}{4\pi R^2\epsilon_0} = \frac{\lambda \ln R}{2\pi\epsilon_0} - \frac{\lambda}{4\pi\epsilon_0}$$

Ytre løsningen er då

$$\phi_o(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{R}\right) - \frac{\lambda}{4\pi\epsilon_0}$$

og indre

$$\phi_i(r) = -\frac{\lambda r^2}{4\pi R^2\epsilon_0}$$



$\rho = \frac{A}{r}$

Kúlaskel með geisla $a < R$

og $\rho(r) = \frac{A}{r}$

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\phi(r)\right) = \frac{A}{r\epsilon_0}$$

$$\frac{d}{dr}\left(r^2\frac{d}{dr}\phi(r)\right) = -\frac{Ar}{\epsilon_0}$$

$$\begin{cases} \dfrac{d}{dr}\phi_{sh}(r) = -\dfrac{A}{2\epsilon_0} + \dfrac{C_1}{r^2} & a<r<R \\[2ex] \dfrac{d}{dr}\phi_0(r) = \dfrac{k_1}{r^2} & r>R \end{cases}$$

$$\frac{d}{dr}\phi_i(r) = \frac{\kappa_1}{r^2} \qquad r < a$$

$\kappa_1 = 0$ því inni í holi er engin hleðsla

samfella í $r = a$

$$-\frac{A}{2\epsilon_0} + \frac{C_1}{a^2} = 0 \quad \rightarrow \quad C_1 = \frac{Aa^2}{2\epsilon_0}$$

samfella í $r = R$

$$\frac{k_1}{R^2} = -\frac{A}{2\epsilon_0} + \frac{C_1}{R^2} = -\frac{A}{2\epsilon_0} + \frac{Aa^2}{2\epsilon_0 R^2}$$

$$\rightarrow k_1 = \frac{Aa^2}{2\epsilon_0} - \frac{AR^2}{2\epsilon_0} = \frac{A}{2\epsilon_0}(a^2 - R^2)$$

Höfum þá

$$\frac{d}{dr}\phi_i(r) = 0 \qquad r<a$$

$$\frac{d}{dr}\phi_{sh}(r) = -\frac{A}{2\epsilon_0} + \frac{A}{2\epsilon_0}\frac{a^2}{r^2} \qquad a<r<R$$

$$\frac{d}{dr}\phi_o(r) = \frac{A}{2\epsilon_0}(a^2-R^2)\frac{1}{r^2} \qquad r>R$$

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heilda

$$\phi_i(r) = K_2 \qquad r<a$$

$$\phi_{sh}(r) = \frac{A}{2\epsilon_0}\left(-\frac{a^2}{r} - r\right) + C_2 \qquad a<r<R$$

$$\phi_o(r) = -\frac{A}{2\epsilon_0}(a^2-R^2)\frac{1}{r} + k_2 \qquad r>R$$

$$\Phi_0(r) = \frac{A}{2\epsilon_0}(R^2 - a^2)\frac{1}{r} + k_2$$

Velgjam nullpunkt
$k_2 = 0$

$$\Phi_{sk}(r) = \frac{A}{2\epsilon_0}(-r^2 - a^2)\frac{1}{r} + C_2$$

Samfella i $r = R$

$$\frac{A}{2\epsilon_0}(R^2 - a^2)\frac{1}{R} = -\frac{A}{2\epsilon_0}(R^2 + a^2)\frac{1}{R} + C_2$$

$$\rightarrow C_2 = \frac{2A}{2\epsilon_0} R^2/R = \frac{AR}{\epsilon_0}$$

Samfella i $r = a$

$$-\frac{A}{2\epsilon_0} 2a^2/a + \frac{AR}{\epsilon_0} = K_2$$

$$\rightarrow \quad K_2 = \frac{A}{\epsilon_0}(R-a)$$

$$\boxed{\begin{aligned} \phi_i(r) &= \frac{A}{\epsilon_0}(R-a) \quad \text{fasti} \\ \phi_{sh}(r) &= -\frac{A}{2\epsilon_0}(r^2+a^2)\frac{1}{r} + \frac{AR}{\epsilon_0} \\ \phi_o(r) &= \frac{A}{2\epsilon_0}(R^2-a^2)\frac{1}{r} \end{aligned}}$$

$r<a$

$a<r<R$

$r>R$

heilder[illegible]lan er $Q = \frac{4\pi A}{2}(R^2-a^2)$

$$\rightarrow \quad \phi_o(r) = \frac{Q}{4\pi\epsilon_0 r}$$

eins og buast mei vid