Pólhnit:

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$
$$\mathbf{a} = \dot{\mathbf{v}} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

Sívalningshnit:

$$x = r\cos\phi,\ y = r\sin\phi,\ z = z$$
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\phi}\mathbf{e}_\phi + \dot{z}\mathbf{e}_z$$

Kúluhnit:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$
$$dv = r^2\sin\theta dr d\theta d\phi$$
$$\mathbf{v} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e}_\theta + r\sin\theta\dot{\phi}\mathbf{e}_\phi$$

Vigrar:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

Kraftlögmál:

$$\mathbf{p} = m\mathbf{v},\ \ \mathbf{F} = \frac{d\mathbf{p}}{dt},\ \ \Delta\mathbf{p} = \int \mathbf{F}dt$$
$$T = \frac{1}{2}mv^2,\ \ \mathbf{F}\cdot\mathbf{v} = \frac{dT}{dt}$$
$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r},\ \ \mathbf{L} = \mathbf{r} \times \mathbf{p},\ \ \dot{\mathbf{L}} = \mathbf{r} \times \mathbf{F}$$

Geymið kraftsvið:

$$\mathbf{F} = -\nabla U,\ \ \oint \mathbf{F}\cdot d\mathbf{r} = 0$$
$$E = T + U = \text{fasti}$$

Sveiflur:

$$F(x) = m\ddot{x} = -kx,$$
$$\omega_0 = \sqrt{k/m},\ \ \tau_0 = \frac{2\pi}{\omega_0}$$
$$x = a\cos(\omega_0 t - \phi)$$

eða

$$x = B\cos(\omega_0 t) + C\sin(\omega_0 t)$$

Deyfing:

$$\beta = b/(2m)$$
$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

með lausn:

$$x = Ae^{-\beta t}\cos(\omega_1 t - \delta)$$

ef $\omega_1^2 = \omega_0^2 - \beta^2 > 0$ (vandeyfing)

Þvinguð sveifla:

$$x = \frac{(F_0/m)}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}\cos(\omega t - \delta)$$

Fourier-röð:

$$F(t) = a_0/2 + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$$

með (τ er lotan):

$$a_n = \frac{2}{\tau}\int_0^\tau F(t)\cos(n\omega t)dt$$
$$b_n = \frac{2}{\tau}\int_0^\tau F(t)\sin(n\omega t)dt$$

Pendúll:

$$\ddot{\theta} + \omega_0^2\sin\theta = 0,\ \ \omega_0^2 = g/l$$

Þyngdarsvið:

$$\mathbf{F} = -\frac{Gm_1m_2}{r^2}\mathbf{e_r},\ \ \mathbf{g} = \frac{\mathbf{F}}{m} = -\nabla\Phi$$

þar sem:

$$\Phi = -\frac{Gm_2}{r},\ \ \Phi = -G\int\frac{\rho}{r}dv.\ \ U = m\Phi$$

Lögmál Gauß:

$$\int_S \mathbf{g}\cdot\mathbf{n}da = -4\pi G\int_V \rho dv,\ \ \nabla\cdot\mathbf{g} = -4\pi G\rho$$

Jafna Poissons:

$$\nabla^2\Phi = 4\pi G\rho$$

Jafna Eulers:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$$

Fall Lagrange:

$$L(q_j, \dot{q}_j, t) = T - U, \quad \delta \int_1^2 L dt = 0$$

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$$

$$p_j = \frac{\partial L}{\partial \dot{q}_j}$$

Fall Hamiltons:

$$H = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$$

$$H = H(q_i, p_i, t)$$

Jöfnur Hamiltons:

$$\frac{\partial H}{\partial p_k} = \dot{q}_k, \quad \frac{\partial H}{\partial q_k} = -\dot{p}_k$$

Geymið kerfi:

$$\frac{\partial H}{\partial t} = 0, \quad H = T + U$$

Miðlægir kraftar:

$$\mathbf{F} = -\frac{\partial U(r)}{\partial r}\mathbf{e_r}, \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2}$$

$$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$$

$$\left(\frac{d^2u}{d\theta^2}\right) + u = -\frac{\mu}{\ell^2}\frac{F(1/u)}{u^2}$$

$$u = 1/r, \quad \ell = \mu r^2 \dot{\theta}$$

$$\theta(r) = \int \frac{(l/r^2)dr}{\sqrt{2\mu\left(E + \frac{k}{r} - \frac{l^2}{2\mu r^2}\right)}} + \text{fasti}$$

Reikistjörnur/gervitungl:

$$F(r) = -\frac{k}{r^2}, \quad r = \frac{\alpha}{1 + \epsilon\cos\theta}$$

Stöðug hringbraut ef:

$$\frac{F'(\rho)}{F(\rho)} > -\frac{3}{\rho}$$

Safn agna:

$$M\mathbf{R} = \sum_\alpha m_\alpha \mathbf{r}_\alpha, \quad M\mathbf{R} = \int \mathbf{r}\rho dv$$

$$\mathbf{F} = M\ddot{\mathbf{R}}, \ \mathbf{N} = \sum_\alpha \mathbf{r}_\alpha \times \mathbf{F}_\alpha$$

$$\mathbf{L} = \sum_\alpha \mathbf{r}_\alpha \times m_\alpha \mathbf{v}_\alpha, \quad \mathbf{N} = \dot{\mathbf{L}}$$

$$T = \frac{1}{2}M\dot{\mathbf{R}}^2 + \frac{1}{2}I_{cm}\omega^2$$

Árekstrar:

$$\sum \mathbf{p} = \text{fasti}, \quad T_{\text{eftir}} = T_{\text{fyrir}} + Q$$

$$\epsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|}, \ \tan\psi = \frac{\sin\theta}{\cos\theta + (m_1/m_2)}$$

Viðmiðunarkerfi:

$$\mathbf{r}' = \mathbf{R} + \mathbf{r}$$

$$\left(\frac{d\mathbf{Q}}{dt}\right)_{K'} = \left(\frac{d\mathbf{Q}}{dt}\right)_K + \boldsymbol{\omega} \times \mathbf{Q}$$

$$\mathbf{a}_{\text{eff}} = \mathbf{a} - \ddot{\mathbf{R}} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) - 2\boldsymbol{\omega}\times\mathbf{v}_r - \dot{\boldsymbol{\omega}}\times\mathbf{r}$$

Stjarfhlutir:

$$I_{ij} = \sum_\alpha m_\alpha (r_\alpha^2 - x_{j\alpha}^2), \quad \text{ef i=j}$$

$$I_{ij} = -\sum_\alpha m_\alpha x_{i\alpha} x_{j\alpha}, \quad \text{ef i}\neq \text{j}$$

$$I_{ij} = \int_V \rho(\mathbf{r})\left(\delta_{ij}\sum_k x_k^2 - x_i x_j\right) dv$$

$$\mathbf{L} = \{\mathbf{I}\} \cdot \boldsymbol{\omega}$$

$$T_{\text{rot}} = \frac{1}{2}\boldsymbol{\omega}\cdot\{\mathbf{I}\}\cdot\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\omega}\cdot\mathbf{L} = \frac{1}{2}\sum_{i,j} I_{ij}\omega_i\omega_j$$

Regla Steiners:

$$I_{jj,cm} = I_{jj} - Ma^2$$

Meginásar - höfuðásar:

$$|\mathbf{I} - I\mathbf{1}| = 0, \quad \mathbf{I}' = \boldsymbol{\lambda}\mathbf{I}\boldsymbol{\lambda}^t, \quad \mathbf{I}' = \boldsymbol{\lambda}\mathbf{I}\boldsymbol{\lambda}^{-1}$$

Horn Eulers:

ϕ um z', θ um x'', ψ um z

$$\omega = (\dot\phi\sin\theta\sin\psi+\dot\theta\cos\psi, \dot\phi\sin\theta\cos\psi-\dot\theta\sin\psi, \dot\phi\cos\theta+\dot\psi)$$

Jöfnur Eulers:

$$I_1\dot\omega_1 - (I_2 - I_3)\omega_2\omega_3 = N_1$$

$$I_2\dot\omega_2 - (I_3 - I_1)\omega_3\omega_1 = N_2$$

$$I_3\dot\omega_3 - (I_1 - I_2)\omega_1\omega_2 = N_3$$

$I_1 = I_2$ ef x_3 er samhverfuás og
$\mathbf{N} = \mathbf{0}$ gefur $\Omega = (I_3 - I_{12})\omega_3/I_{12}$

Snúður:

$p_\psi = I_3\omega_3 = \text{fasti}$, $p_\phi = \text{fasti}$

$$E = \frac{1}{2}I_{12}(\dot\phi\sin^2\theta+\dot\theta^2)+\frac{1}{2}I_3\omega_3^2+Mgh\cos\theta = \text{fasti}$$

Pólvelta:

$\dot\phi_{0(-)} \approx Mgh/I_3\omega_3$

$\dot\phi_{0(+)} \approx I_3\omega_3/(I_1\cos\theta_0)$

Tengdar sveiflur um jafnvægisstöðu:

$$T = \frac{1}{2}\sum_{j,k} m_{jk}\dot q_j \dot q_k$$

$$U = \frac{1}{2}\sum_{j,k} A_{jk} q_j q_k$$

öll q cos-föll af ωt ef $|A_{jk} - \omega^2 m_{jk}| = 0$, n rætur, hlutföll hnitanna $a_1 : a_2 : a_3$ finnast út úr $\sum_j (A_{jk} - \omega^2 m_{jk})a_j = 0$

$$q_j(t) = \sum_r a_{jr}\cos(\omega_r t - \delta_r)$$

skv. upphafsskilyrðum.

$$\{\mathbf{A} - \omega^2\mathbf{M}\}\mathbf{a} = 0$$

$$\mathbf{U}^t\mathbf{A}\mathbf{U} = \mathbf{A}_{\text{diag}}, \quad \mathbf{U}^t\mathbf{M}\mathbf{U} = 1$$

Örfáar jöfnur úr stærðfræði

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$\int dx \ln(a+bx) = \left(x + \frac{a}{b}\right)\ln(a+bx) - x$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$(1 \pm x)^{\frac{1}{2}} = 1 \pm \frac{1}{2}x - \frac{1}{8}x^2 \pm \frac{1}{16}x^3 + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$