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Lárétt hreyfing - mótstöðukraftur $F = -\gamma v^3$

$t = 0$

$v_0 \;\bullet\!\!\longrightarrow v(t),\ x(t)\,?$

$x = 0$

Finna ferðjöfnur, Hreyfijafna $m\ddot{x} = -\gamma v^3$ eða $m\dot{v} = -\gamma v^3$

$$m\frac{dv}{dt} = -\gamma v^3 \quad \longrightarrow \quad \int_{v_0}^{v} \frac{dv'}{v'^3} = -\frac{\gamma}{m}\int_0^t dt'$$

$$\longrightarrow \quad -\frac{1}{2v^2} + \frac{1}{2v_0^2} = -\frac{\gamma}{m}(t-0) \quad \longrightarrow \quad -\frac{1}{2v^2} = -\frac{\gamma t}{m} - \frac{1}{2v_0^2}$$

$$\longrightarrow \quad 2v^2 = \frac{1}{\frac{\gamma t}{m} + \frac{1}{2v_0^2}} = \frac{m}{\gamma t + \frac{m}{2v_0^2}}$$

$$v^2 = \frac{m v_0^2}{2\gamma v_0^2 t + m} \quad \rightarrow \quad \boxed{v = v_0 \sqrt{\frac{m}{2\gamma v_0^2 t + m}}}$$

$$\frac{dx}{dt} = v_0 \sqrt{\frac{m}{2\gamma v_0^2 t + m}} \quad \rightarrow \quad dx = v_0\, dt \sqrt{\frac{m}{2\gamma v_0^2 t + m}}$$

$$\int_0^x dx' = v_0 \int_0^t dt' \sqrt{\frac{m}{2\gamma v_0^2 t + m}} = \frac{2 v_0 \sqrt{m}}{2\gamma v_0^2} \left\{ \sqrt{2\gamma v_0^2 t + m} - \sqrt{m} \right\}$$

$$= \frac{\sqrt{2\gamma v_0^2 t m + m^2}}{\gamma v_0} - \frac{m}{\gamma v_0} = x(t)$$

$$\boxed{x(t) = \sqrt{\frac{2tm}{\gamma} + \frac{m^2}{\gamma^2 v_0^2}} - \frac{m}{\gamma v_0}}$$

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Ågått ~~er det store fasta~~ $v(0) = v_0 \cdot 1$

og $x(0) = \sqrt{\frac{m^2}{\gamma^2 v_0^2}} - \frac{m}{\gamma v_0} = 0$

Vidder gening

$$\underbrace{m\frac{dv}{dt}} = -\gamma v^3$$

$$\frac{ML}{T^2} = [\gamma]\frac{L^3}{T^3} \rightarrow [\gamma] = M\frac{T}{L^2}$$

$$[\gamma v_0^2 t] = M\frac{T}{L^2}\frac{L^2}{T^2}T = M = [m]$$ fyrir $v = v_0\sqrt{\frac{m}{2\gamma v_0^2 t + m}}$

$$\left[\frac{tm}{\gamma}\right] = \frac{TML^2}{MT} = L^2 = \left[\frac{m^2}{\gamma^2 v_0^2}\right] = \frac{M^2 L^4 T^2}{M^2 T^2 L^2} = L^2$$ fyr $x(t) = \cdots$

(6)

Kraftur $F = -\frac{mk^2}{x^3}$ $\quad k = $ fasti

a) Stöðuorka

$$F = -\frac{\partial U}{\partial x} \quad \rightarrow \quad U = -\frac{mk^2}{2x^2}$$ kemur til greina

ef við setjum $U = 0$ þegar $x \rightarrow \infty$

b) Hlutlægur kraftur $\rightarrow$ E er fasti

Kyrrstaða ögu $\rightarrow$ í fjarlægð d frá miðju sviðs

er orkan $E = U(d) = -\frac{mk^2}{2d^2}$

$$E = \frac{m}{2}\dot{x}^2 - \frac{mk^2}{2}\frac{1}{x^2} = -\frac{mk^2}{2d^2}$$

Notum hér að $\dot{x} = \frac{dx}{dt}$

$$\frac{m}{2}\dot{x}^2 = \frac{mk^2}{2}\left[\frac{1}{x^2} - \frac{1}{d^2}\right] \rightarrow \left(\frac{dx}{dt}\right)^2 = k^2\left[\frac{1}{x^2} - \frac{1}{d^2}\right]$$

$$\rightarrow \frac{dx}{dt} = \pm \frac{k}{\sqrt{\frac{1}{x^2} - \frac{1}{d^2}}} \rightarrow dt = \frac{\pm dx}{k\sqrt{\frac{1}{x^2} - \frac{1}{d^2}}}$$

$$\rightarrow \int_0^t dt' = \pm \frac{1}{k}\int_d^0 \frac{dx}{\sqrt{\frac{1}{x^2} - \frac{1}{d^2}}} = \pm \frac{d}{k}\int_d^0 \frac{x\,dx}{\sqrt{d^2 - x^2}}$$

(E.9)

$$= \pm \frac{d}{k}\left\{-\sqrt{d^2 - x^2}\,\Big|_d^0\right\} + 0 = \mp \frac{d^2}{k}$$

$$\rightarrow t = \mp \frac{d^2}{k}$$

með því að velja neikvæðu greinina fyrir kvaðratrótina

fæst eðlisfræðilega ásættanleg lausn

$$\boxed{t = \frac{d^2}{k}}$$

7 Lausnin er að mestu leiti í Example 7.12 í bók

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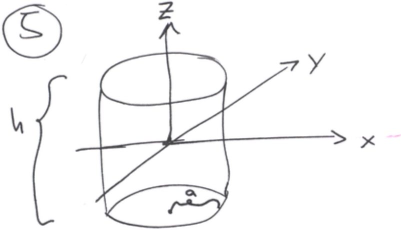

$$I_{ij} = \int_V dv\, \rho(\vec{r}) \left\{ \delta_{ij} \sum_k x_k^2 - x_i x_j \right\}$$

Byrjum með

$$I_{33} = \rho \int_0^a r\,dr \left\{ x^2 + y^2 \right\} \int_0^{2\pi} d\phi \int_{-\frac{h}{2}}^{\frac{h}{2}} dz$$

$$= \rho\, 2\pi h \int_0^a r^3 dr = \rho\, 2\pi h \frac{a^4}{4} = \frac{M\, 2\pi h\, a^4}{\pi a^2 h \cdot 4}$$

$$= \frac{M}{2} a^2$$

$$\rho = \frac{M}{V} = \frac{M}{\pi a^2 h}$$

$$I_{11} = \rho \int_0^a r\,dr \int_0^{2\pi} d\phi \int_{-\frac{h}{2}}^{\frac{h}{2}} dz \left\{ y^2 + z^2 \right\}$$

$$= \rho \int_0^a r\,dr \int_0^{2\pi} d\phi \int_{-h/2}^{h/2} dz \left\{ (r\sin\phi)^2 + z^2 \right\}$$

$$= \rho \int_0^a r\,dr \int_0^{2\pi} d\phi \left\{ h r^2 \sin^2\phi + \frac{h^3}{12} \right\}$$

$$= \rho \int_0^a r\,dr \left\{ \pi \cdot h r^2 + 2\pi \frac{h^3}{12} \right\} = \rho \left\{ \frac{h a^4 \pi}{4} + 2\pi \frac{a}{2} \frac{h^3}{12} \right\}$$

$$I_{11} = \frac{M}{\pi a^2 h}\left\{\frac{h a^4 \pi}{4} + \frac{2\pi}{2} a \frac{h^3}{12}\right\} = \frac{1}{12} M \left\{3a^2 + h^2\right\}$$

Nákvæmlega það sama fæst fyrir $I_{22} = I_{11}$
því heildið yfir $\cos^2\phi$ gefur það sama og $\sin^2\phi$

Hnitakerfið er sett upp um samhverfuása sívalnings
því fæst að þetta eru höfuðásarnir

$$\mathbb{I} = \begin{pmatrix} \frac{M}{12}(3a^2+h^2) & 0 & 0 \\ 0 & \frac{M}{12}(3a^2+h^2) & 0 \\ 0 & 0 & \frac{M}{2}a^2 \end{pmatrix}$$