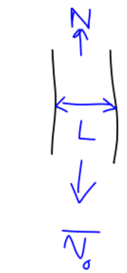
1

Dæmi 1

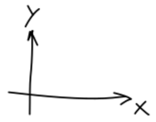


Notum DC (12.36)

$$\bar F_r^{eff} = \bar F - m\bar a_f - m2\bar\omega\times\bar v_f - m\bar\omega\times(\bar\omega\times\bar r_r) - m\dot{\bar\omega}\times\bar r_r$$

eingungis krafur Coriolis skiptir máli hér

$$\bar v = -v_0\hat e_y$$



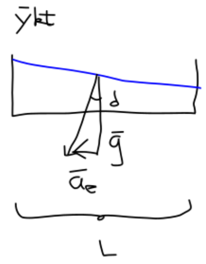
Hnitakerfi samkvæmt mynd 12.7 í DC, kerfi sem snýst með jörðinni

$$\bar\omega = 0\hat e_x + \omega\cos\lambda\,\hat e_y + \omega\sin\lambda\,\hat e_z$$

$$\to \bar a_c = 2\{-v_0\hat e_y\}\times\{\hat e_z\}\,\omega\sin\lambda = -2v_0\omega\sin\lambda\,\hat e_x$$

--> hröðun í átt að vesturbakka skurðsins

2



Yfirborð vatns í stöðugu ástandi verður alltaf þvert á virka þyngdarkraftinn á það. Köllum hnikunarhornið δ

$$\sin\delta = \frac{a_c}{\sqrt{g^2 + a_c^2}} = \frac{2v_0\omega\sin\lambda}{\sqrt{g^2 + (2v_0\omega\sin\lambda)^2}}$$

því má fá fyrir hæðarmun vatnsborðs við bakkana

$$\Delta h = L\sin\delta = \frac{2v_0 L\omega\sin\lambda}{\sqrt{g^2 + (2v_0\omega\sin\lambda)^2}}$$

$\omega = 7{,}3\cdot10^{-5}$ rad/s

$g = 9{,}82$ m/s² $\quad L = 50$ m

$v_0 = 12$ m/s $\quad \lambda = \frac{65^\circ\pi}{180}$

$= 0{,}00808$ m $= 8{,}08$ mm

3

Dæmi 2

Notum jöfnur (12.65-67) til að kanna stærðargráður fyrir kúlu skotinni beint í austur með $v_0 = 200$ m/s

$\omega = 7{,}3\cdot10^{-5}$ 1/s

$g = 9{,}82$ m/s²

$\lambda = \frac{65\pi}{180}$

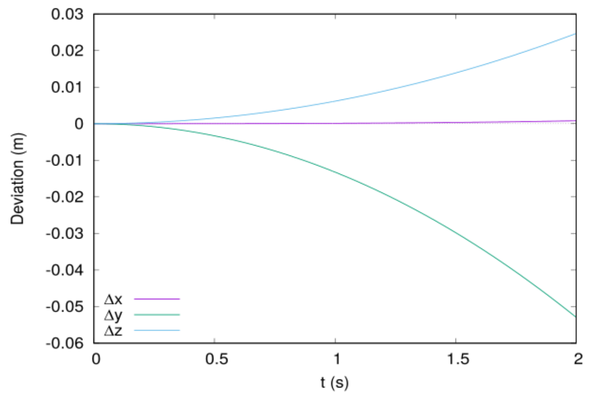
$\dot z_0' = 0$

$\dot y_0' = 0$

$$\Delta x' = \frac{\omega g t^3}{3}\cos\lambda - \dot x_0' t$$

$$\Delta y' = -\omega\dot x_0' t^2\sin\lambda$$

$$\Delta z' = \omega\dot x_0' t^2\cos\lambda$$



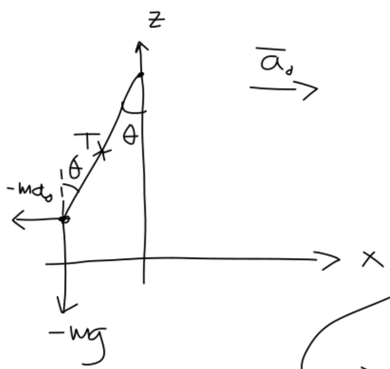
Það gleymist oft að kraftur Coriolis veldur ekki bara hægri beygju, heldur líka þætti upp á við í þessum aðstæðum

4

Dæmi 3

$$\bar F_{eff} = \bar F - m\ddot{\bar R}_f - m\dot{\bar\omega}\times r - m\bar\omega\times(\bar\omega\times\bar r) - 2m\bar\omega\times\bar v_r$$

a) Föst hröðun $\bar a_0$
finna T og útslag



$$\bar F_{eff} = -g\hat e_z m - ma_0\hat e_x$$
$$= m(-a_0, 0, -g)$$

(z): $T\cos\theta = mg$

(x): $T\sin\theta = ma_0$

$$\tan\theta = \frac{a_0}{g}$$

$\to \theta = 0$ ef $a_0 = 0$

5

Eins sést að

$$T^2\cos^2\theta + T^2\sin^2\theta = (mg)^2 + (ma_0)^2$$

$$\to \quad T^2 = (mg)^2 + (ma_0)^2$$

$$\to \quad \boxed{T = m\sqrt{g^2 + a_0^2}}$$

og áður

$$\boxed{\theta = \arctan\left(\frac{a_0}{g}\right)}$$

b) Hringferð með geisla R og fastri ferð v_0

$$\bar{F}_{eff} = \bar{g} - m\bar{\omega}\times(\bar{\omega}\times\bar{r})$$

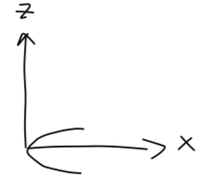
Hnitakerfið snýst með farartækinu

$$\to \quad \ddot{\bar{R}}_f = 0, \quad \bar{v}_r = 0$$

6

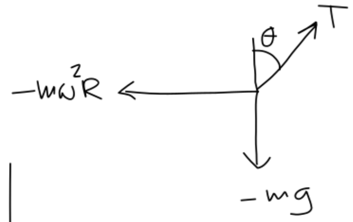
$$\bar{\omega} = \omega\hat{e}_z$$

$$\bar{F}_{eff} = -mg\hat{e}_z - m\omega^2 R\,\hat{e}_x$$



$$T\cos\theta = mg$$

$$T\sin\theta = m\omega^2 R = m\frac{v_0^2}{R}$$



$$\to \quad \tan\theta = \frac{mv_0^2}{Rmg} = \frac{v_0^2}{Rg}$$

$$\to \quad \boxed{\theta = \arctan\left(\frac{v_0^2}{Rg}\right)}$$

$$T^2 = (mg)^2 + \left(m\frac{v_0^2}{R}\right)^2$$

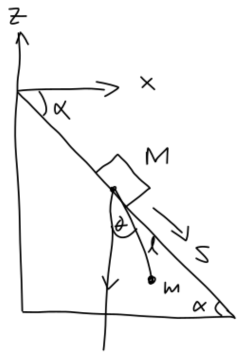
$$\to \quad \boxed{T = m\sqrt{g^2 + \frac{v_0^4}{R^2}}}$$

$$v_0 = R\omega$$

$$\to \quad \omega = \frac{v_0}{R}$$

7

Dæmi 4



Fyrir M: $\begin{cases} x = s\cos\alpha \\ z = -s\sin\alpha \end{cases}$

Hnit massa

fyrir m: $\begin{cases} x = s\cos\alpha + l\sin\theta \\ z = -s\sin\alpha - l\cos\theta \end{cases}$

Alhraðar

$$v_M = \dot{s}$$

$$v_m = (\dot{s}\cos\alpha + l\dot{\theta}\cos\theta,\; -\dot{s}\sin\alpha + l\dot{\theta}\sin\theta)$$

$$\to \quad v_m^2 = \dot{s}^2 + (l\dot{\theta})^2 + 2l\dot{s}\dot{\theta}\left[\cos\alpha\cos\theta - \sin\alpha\sin\theta\right]$$

$$= \dot{s}^2 + (l\dot{\theta})^2 + 2l\dot{s}\dot{\theta}\cos(\theta+\alpha)$$

8

$$T = \frac{M}{2}v_M^2 + \frac{m}{2}v_m^2, \quad U = Mgz_M + mgz_m$$

$$\to \quad \boxed{L = \frac{M}{2}\dot{s}^2 + \frac{m}{2}\left[\dot{s}^2 + (l\dot{\theta})^2 + 2l\dot{s}\dot{\theta}\cos(\theta+\alpha)\right] + Mgs\sin\alpha + mg\left[s\sin\alpha + l\cos\theta\right]}$$

Tvö alhnit, s og θ

$$\frac{\partial L}{\partial s} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) = 0$$

$$Mg\sin\alpha + mg\sin\alpha - \frac{d}{dt}\left[M\dot{s} + m\dot{s} + ml\dot{\theta}\cos(\theta+\alpha)\right] = 0$$

$$\to \quad g(M+m)\sin\alpha - (M+m)\ddot{s} - ml\ddot{\theta}\cos(\theta+\alpha) + ml\dot{\theta}^2\sin(\theta+\alpha) = 0$$

(9)

$$\boxed{(M+m)\ddot{s} + ml\{\ddot{\theta}\cos(\theta+\alpha) - \dot{\theta}^2\sin(\theta+\alpha)\} - g(M+m)\sin\alpha = 0}$$

$$\frac{\partial L}{\partial\theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = 0$$

$$-ml\dot{s}\dot{\theta}\sin(\theta+\alpha) - mgl\sin\theta - \frac{d}{dt}\left[ml\dot{s}\cos(\theta+\alpha) - ml^2\dot{\theta}\right] = 0$$

$$\rightarrow \cancel{-ml\dot{s}\dot{\theta}\sin(\theta+\alpha)} - mgl\sin\theta - ml\ddot{s}\cos(\theta+\alpha) + \cancel{ml\dot{s}\dot{\theta}\sin(\theta+\alpha)} - ml^2\ddot{\theta} = 0$$

$$\rightarrow ml^2\ddot{\theta} + ml\ddot{s}\cos(\theta+\alpha) + mgl\sin\theta = 0$$

$$\rightarrow \boxed{l\ddot{\theta} + \ddot{s}\cos(\theta+\alpha) + g\sin\theta = 0}$$

(10)

Smáar sveiflur, fyrst jafnvægisstaða? $\rightarrow \ddot{\theta} = 0, \; \dot{\theta} = 0$

(1) $\rightarrow \boxed{\ddot{s} - g\sin\alpha \simeq 0}$

(2) $\rightarrow \boxed{\ddot{s}\cos(\theta+\alpha) + g\sin\theta \simeq 0}$

$$\rightarrow g\sin\alpha\cos(\theta+\alpha) + g\sin\theta = 0$$

$$\rightarrow \theta_0 = -\alpha \quad (\text{og } \theta_0 = \pi - \alpha) \leftarrow \text{óstöðug}$$

$$\rightarrow \theta = \theta_0 + \delta = -\alpha + \delta$$

til að kanna smáar sveiflur

(11)

Setjum inn í hreyfijöfnur

$$(M+m)\ddot{s} + ml[\ddot{\delta}\cos(\delta) - \dot{\delta}^2\sin(\delta)] - g(M+m)\sin\alpha = 0$$

$$\rightarrow \boxed{(M+m)\ddot{s} + ml\left\{\ddot{\delta}\left(1-\frac{\delta^2}{2}\right) - \dot{\delta}^2\left(\delta - \frac{\delta^3}{6}\right)\right\} - g(M+m)\sin\alpha = 0}$$

$$l\ddot{\delta} + \ddot{s}\left(1-\frac{\delta^2}{2}\right) + g\sin(-\alpha+\delta) = 0$$

$$\rightarrow l\ddot{\delta} + \ddot{s}\left(1-\frac{\delta^2}{2}\right) + g\{-\sin\alpha\cos\delta + \cos\alpha\sin\delta\} = 0$$

eða línulega

$$\boxed{l\ddot{\delta} + \ddot{s} - g\sin\alpha + g\cos\alpha\cdot\delta = 0}$$

$$\boxed{(M+m)\ddot{s} + ml\ddot{\delta} - g(M+m)\sin\alpha = 0}$$

(12)

Útbúum nýja breytu

$$\ddot{z} \equiv \ddot{s} - g\sin\alpha$$

til að fá línulega afleiðujöfnuhneppið

$$\boxed{\begin{aligned} l\ddot{\delta} + \ddot{z} + g\cos\alpha\cdot\delta &= 0 \\ (M+m)\ddot{z} + ml\ddot{\delta} &= 0 \end{aligned}}$$

Þetta er annarsstigs afleiðuhneppi, og við lærum í síðasta hluta námskeiðsins að eiga við þau, en hér nýtum við okkur að þetta er einfalt vegna seinni jöfnunar

$$\rightarrow l\ddot{\delta} - \frac{ml}{M+m}\ddot{\delta} + g\cos\alpha\cdot\delta = 0$$

$$\boxed{\ddot{\delta} + \frac{g}{l}\frac{\cos\alpha}{\left(\frac{M}{M+m}\right)}\delta = 0}$$

$$\boxed{\omega = \sqrt{\frac{g\cos\alpha}{l}\,\frac{M+m}{M}} = \sqrt{\frac{g\cos\alpha}{l}\left(1+\frac{m}{M}\right)}}$$

svo er í raun annar sveifluháttur, en vegna útlits seinni jöfnunar er þar $\omega = 0$, skoðum aftur seinna