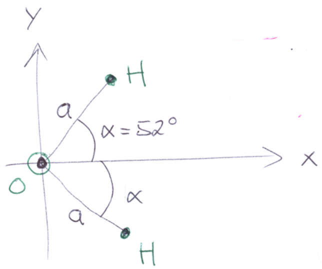
09-07 (1)

H_2O , finna CM



spegilsamhverfa um x-ás

$\rightarrow Y_{CM} = 0$

$$X_{CM} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i$$

$m_O = 16 m_H$

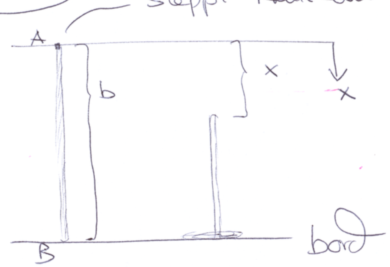
$$M = m_O + 2m_H = (16+2)m_H$$

$$X_{CM} = \frac{1}{18 m_H}\cdot 2m_H \cdot a \cdot \cos\alpha$$

$$= \frac{a}{9}\cos\alpha \approx 0.068 \cdot a$$

09-19 (2)

sleppt keðjan $t=0$, lengd b



fyrir $t>0$

$$\rho = \frac{M}{b}$$

Hver er kraftur borðsins F á keðjuna sem fall af x?

Þegar hún hefur öll fallið er $\vec{F} = +\rho b g \hat{e}_x$ og massamiðjan kyrr $\rightarrow \frac{dp}{dt} = 0$, því kraftur borðsins er $-\vec{F}$

því er $\frac{dp}{dt} = \rho b g - F$ $\qquad p = \rho(b-x)\dot{x}$

(3)

$$\frac{dp}{dt} = \frac{d}{dt}\{\rho(b-x)\dot{x}\} = -\rho\dot{x}\dot{x} + \rho(b-x)\ddot{x}$$

Massamiðjan er í þyngdarsviði

$$\rightarrow \boxed{\ddot{x} - g = 0}$$

fellur úr kyrrstöðu með $x=0$

$\rightarrow E = 0$

Orkuvarðveisla $E = \frac{m}{2}\dot{x}^2 - mgx$,

$$\rightarrow \boxed{\dot{x}^2 = 2gx}$$

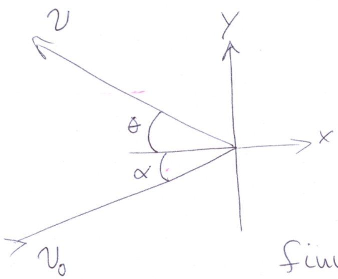
$$\frac{dp}{dt} = -\rho 2gx + \rho(b-x)g = \rho\{bg - 3gx\}$$

og áður

$$\frac{dp}{dt} = \rho b g - F \rightarrow \boxed{F = 3\rho g x}$$

og $\vec{F} = -3\rho g x \hat{e}_x$

09-42 (4)



$\alpha = 30°$

$v_0 = 5$ m/s

$$\epsilon = \frac{|v|}{|v_0|}$$

hér $\epsilon = 0{,}8$

finna v og θ

Engin breyting í hreyfingu í y-átt

$$\rightarrow \underbrace{v_0 \sin\alpha}_{v_{y0}} = \underbrace{v\sin\theta}_{v_y} \qquad \rightarrow v = v_0\frac{\sin\alpha}{\sin\theta}$$

$$v_y = \underbrace{v_0 \sin\alpha}_{\text{þekkt}}$$

Vegna ófjaðrandi árekstrs, lýst með ϵ

$$\rightarrow \left|\frac{v_x}{v_{x0}}\right| = \left|\frac{v\cos\theta}{v_0\cos\alpha}\right| = \epsilon$$

(5)

$$v_x = \epsilon v_0 \cos\alpha$$

$$\to v = \sqrt{v_x^2 + v_y^2} = v_0\sqrt{\epsilon^2\cos^2\alpha + \sin^2\alpha} = v_0 \cdot 0,8544$$

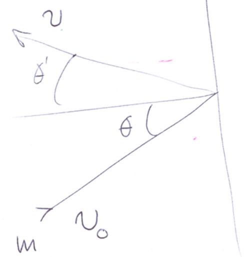
$$\frac{v_y}{v_x} = \frac{v_0 \sin\alpha}{\epsilon v_0 \cos\alpha} = \tan\theta$$

$$\to \theta = \arctan\left\{\frac{\sin\alpha}{\epsilon\cos\alpha}\right\} = 0,62513 \text{ rad}$$

$$\approx 35.82^\circ$$

(6)

09-39



ϵ gefið með θ og m

finna v og θ'

sama dæmi og dæmið á undan, og nú sést hvers vegna ekki var gott að setja strax inn tölur þar

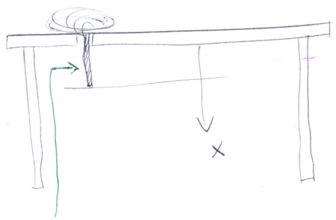
$$v = v_0\sqrt{\epsilon^2\cos^2\theta + \sin^2\theta'}$$

$$\theta' = \arctan\left\{\frac{1}{\epsilon}\tan\theta\right\}$$

(7)

09-15

Klukkan $t=0$ fellur reipið niður um gatið. Ekkert viðnám, finna v og a sem föll af x. Heildar lengd reipis er L



$$\rho = \frac{M}{L}$$

$m = \rho x$ er massi reipis sem hangir

$$\frac{dp}{dt} = mg = g\rho x$$

$$\frac{d}{dt}(mv) = \dot{m}v + m\dot{v} = \rho\dot{x}v + m\dot{v} = \rho v^2 + m\dot{v}$$

$$\rho v^2 + \rho x\dot{v} = g\rho x \to v^2 + x\dot{v} = gx$$

$$\to v^2 + x\frac{dv}{dt} = gx \to v^2 + x\frac{dv}{dx}\frac{dx}{dt} = gx$$

(8)

$$\boxed{v^2 + xv\frac{dv}{dx} = gx}$$

ólínuleg diffurjafna fyrir v sem falli af x (t er horfið)

upphafsskilyrði $v(0)=0$, $x(0)=0$, reynum því veldislausn

$v = cx^\alpha$, setjum inn í jöfnu

$$(cx^\alpha)^2 + x(cx^\alpha)(c\alpha x^{\alpha-1}) = gx$$

$$\to c^2x^{2\alpha} + c^2\alpha x^{2\alpha} = gx$$

$$c^2\{1+\alpha\}x^{2\alpha} = gx$$

$$c^2\{1+\alpha\}x^{2\alpha} = gx$$

því fæst $2\alpha = 1$

og

$$\underbrace{c^2\{1+\alpha\} = g}$$

$$c^2\frac{3}{2} = g$$

$$\to c = \sqrt{\frac{2g}{3}}$$

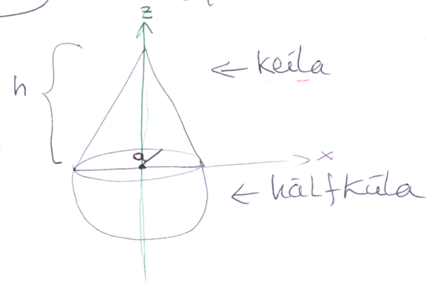
þá er lausnin

$$\boxed{v = \sqrt{\frac{2gx}{3}}}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} = \sqrt{\frac{2gx}{3}}\,\frac{1}{2}\sqrt{\frac{2g}{3x}}$$

$$= \frac{2g}{3}\cdot\frac{1}{2} = \frac{g}{3}$$

09-03 CM fyrir



samhverft um z-ás

→ $y_{cm} = 0$, $x_{cm} = 0$

Verðum að finna z_{cm}

Sívalningur:

$$z_{cm} = \frac{1}{M}\int_A z\,dm$$

$$z_{cm} = \frac{\int_0^{2\pi} d\theta \int_0^a r\,dr \int_0^{h-\frac{hr}{a}} dz\,\rho z}{\int_0^{2\pi} d\theta \int_0^a r\,dr \int_0^{h-\frac{hr}{a}} dz\,\rho}$$

$$z_{cm} = \frac{\frac{h^2}{2}\int_0^a r\,dr\,\left(1-\frac{r}{a}\right)^2}{h\int_0^a r\,dr\,\left(1-\frac{r}{a}\right)} = \frac{\frac{h^2}{2}\,\frac{a^2}{12}}{h\,\frac{a^2}{6}} = h\cdot\frac{1}{4}$$

Hálfkúla:

$z = r\cos\theta$

$$z_{cm} = \frac{\int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \sin\theta\,d\theta \int_0^a r^2\,dr\,\rho z}{\int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \sin\theta\,d\theta \int_0^a r^2\,dr\,\rho}$$

$$z_{cm} = \frac{2\pi\int_{\pi/2}^{\pi} \sin\theta\cos\theta\cdot d\theta \int_0^a r^3\,dr\cdot\rho}{\frac{4\pi}{6}a^3\cdot\rho} = \frac{-\frac{1}{2}\cdot\frac{a^4}{4}\rho}{\frac{1}{3}a^3\rho}$$

$$= -\frac{3}{8}a$$

→ Ef hlutirnir eru með sama massaþéttleikann fast

$$z_{cm} = \frac{m_K (z_{cm})_K + m_{hK}(z_{cm})_{hK}}{m_K + m_{hK}}$$

$$Z_{CM} = \frac{\left(2\pi \frac{ha^2}{6} \rho\right) \cdot \frac{h}{4} - \left(\frac{4\pi}{6} a^3 \rho\right) \frac{3a}{8}}{2\pi \frac{ha^2}{6} \rho + \frac{4\pi}{6} a^3 \rho}$$

$$= \frac{h^2 - 3a^3}{4(h + 2a)}$$