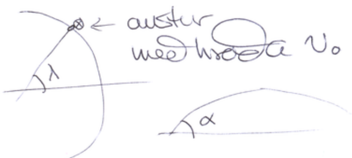
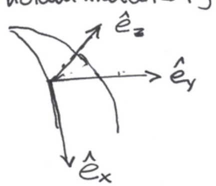
(2) Dæmi 10-09 í bók

Notum $\bar F_{eff} = \bar S + m\bar g - 2m\bar\omega\times\bar v_r$, $\bar S = 0$

$$\to \bar a_r = \bar g - 2\bar\omega\times\bar v_r$$



notum hnitakerfi eins og í Ex. 10.3



$$\bar a_r = -g\hat a_z - 2(-\omega\cos\lambda,\ 0,\ \omega\sin\lambda)\times(0,\ v_y,\ v_z)$$

$$= -g\hat a_z - 2(-v_y\omega\sin\lambda,\ v_z\omega\cos\lambda,\ -v_y\omega\cos\lambda)$$

finna þver geigun, þ.e. í $\hat e_x$-stefnu

skoðum því a_x

$$a_x = 2v_y\omega\sin\lambda = 2v_0\cos\alpha\cdot\omega\cdot\sin\lambda = 2v_0\omega\cos\alpha\sin\lambda$$

setjum upphafsgildi

$\dot x(0) = 0$

$x(0) = 0$

$$a_x = 2v_0\omega\cos\alpha\sin\lambda$$

$$\to v_x(t) = 2v_0\omega t\cos\alpha\sin\lambda$$

$$x(t) = v_0\omega t^2\cos\alpha\cdot\sin\lambda$$

Sem fyrstu nálgun gerum við ráð fyrir að z-hreyfingin sé óháð af krafti Coriolis

Hröðunarjafnan á síðunni að undan sýnir að þetta er vissulega nálgun

$$\to z(t) = v_0 t\sin\alpha - \frac{gt^2}{2}$$

flugtíminn er því

$$T = \frac{2v_0\sin\alpha}{g}$$

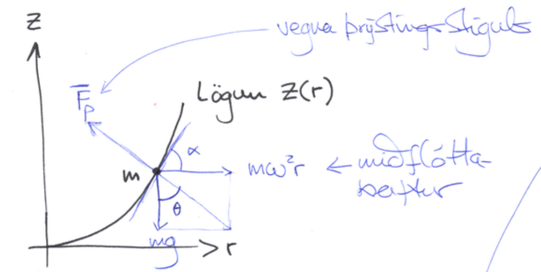
$$x(T) = v_0\omega\left[\frac{2v_0\sin\alpha}{g}\right]^2\cos\alpha\sin\lambda = \frac{4\omega v_0^3}{g^2}\sin\lambda\cos\alpha\sin^2\alpha$$

$$\to d = x(T) - x(0) = \frac{4\omega v_0^3}{g^2}\sin\lambda\cos\alpha\sin^2\alpha$$

athyglisvert að v_0 kemur í 3. veldi!

(3) Dæmi 10-06 í bók

Vatnsfata snýst um snúningsás. Hver er lögun vatnsyfirborðsins?



fyrir m gildir

$$\bar F_{eff} = \bar F - m\ddot{\bar R}_f - m\dot{\bar\omega}\times r - m\bar\omega\times(\bar\omega\times\bar r) - 2m\bar\omega\times\bar v_r$$

Snúningskerfið er með miðju í snúningsás

m hreyfist ekki í því kerfi

$\to \bar F_{eff} = 0$

$\bar v_r = 0$

$\ddot{\bar R}_f = 0$

$\dot{\bar\omega} = 0$

$$0 = \bar F - m\bar\omega\times(\bar\omega\times\bar r)$$

$$\bar F = m\bar g + \bar F_p$$

$\bar F_p$ er hornrétt á yfirborðið ($\bar F_{eff} = 0$), $\to$ $\overrightarrow{(\text{þyngd})} + \overrightarrow{(\text{miðflóttakr})}$ eru líka hornrétt á það

því fæst $\alpha = \theta$

$$\tan\theta = \frac{m\omega^2 r}{mg}$$

og $\frac{dz}{dr} = \tan\theta$

(sjá snertil á mynd)

$$\to \frac{dz}{dr} = \frac{\omega^2 r}{g}$$

$$dz = \frac{\omega^2 r}{g}dr$$

$$\to z = \frac{\omega^2}{2g}r^2 + C$$

heildunarfasti

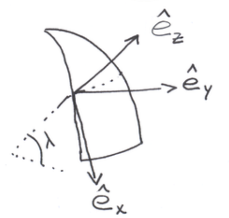
fleygboga lögun

(4) Dæmi 10-08 í bók

Norðurhvel, ögn hent upp í hæð h yfir punkti á yfirborði jarðar, sýna hvar hún lendir (hnikað til vesturs)

sleppa loftmótstöðu, $\frac{h}{a_J} \ll 1$

Hnit eins og á mynd 10-9 í bók



Hröðun vegna krafts Coriolis er

$$\bar a = -2\bar\omega\times\dot{\bar r}$$

(5)

$$\bar\omega \times \dot{\bar r} = \begin{vmatrix} \hat i & \hat j & \hat k \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ 0 & 0 & \dot z \end{vmatrix} = (0,\ \omega\cos\lambda\cdot\dot z,\ 0)$$

því nálgun!

$$\rightarrow \bar a = -2\bar\omega\times\dot{\bar r} = -2\omega\dot z\cos\lambda\ \hat e_y$$

$$\rightarrow \ddot y = -2\omega\dot z\cos\lambda$$

Fyrir z-stefnu

$$z = v_0 t - \frac{gt^2}{2}$$

$$v_z^2 = v_0^2 - 2zg$$

$$\rightarrow v_0 = \sqrt{2gh}$$

heildum í tíma

$$\dot y = -2\omega z\cos\lambda + C_1$$

$$\dot y(0) = 0 \rightarrow C_1 = 0$$

$$\dot y = -2\omega\left\{v_0 t - \frac{gt^2}{2}\right\}\cos\lambda$$

$$= -\omega\cos\lambda\cdot\left\{2v_0 t - gt^2\right\}$$

heildum aftur

$$y = -\omega\cos\lambda\cdot\left\{v_0t^2 - \frac{gt^3}{3}\right\} + C_2$$

(6)

$$y(0) = 0 \rightarrow C_2 = 0$$

$$y = -\omega\cos\lambda\cdot\left\{v_0t^2 - \frac{gt^3}{3}\right\}$$

flugtími: $T = \frac{2v_0}{g}$

$$\rightarrow y = -\omega\cos\lambda\cdot\left\{v_0\left(\frac{2v_0}{g}\right)^2 - \frac{g}{3}\left(\frac{2v_0}{g}\right)^3\right\}$$

$$= -\omega\cos\lambda\cdot\frac{v_0^3}{g^2}\cdot\left\{4 - \frac{8}{3}\right\}$$

$$= -\omega\cos\lambda\cdot\frac{4}{3}\frac{v_0^3}{g^2}$$

$$v_0 = \sqrt{2gh}$$

$$\rightarrow y = -\omega\cos\lambda\cdot\frac{4}{3g^2}(2gh)^{3/2}$$

$$= -\frac{4\omega}{3}\cos\lambda\cdot\sqrt{\frac{8h^3}{g}}$$

í vesturátt

(7)

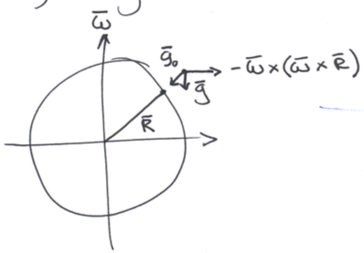
5) Dæmi 10-17 í bók

Stöðuvatn nálgað sem hringur („spherical cow")

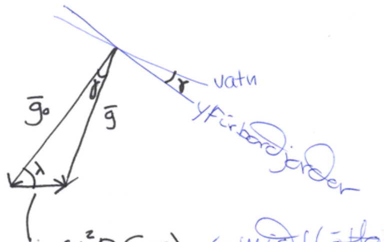
geisli 162 km, $\lambda = 47°$

Finna hve mikið miðsóknar-krafturinn lækkar miðjuna m.v. strönd

Sjá mynd 10-6 í bók



teiknum vigrana aftur



$\omega^2 R\cos\lambda$ ← miðflóttahröðun

$$|g| = \left\{(\omega^2R\cos\lambda)^2 + (g_0)^2 - 2(\omega^2R\cos\lambda)\, g_0\cos\lambda\right\}^{1/2}$$

↑ cosínusregla

$$\frac{|g|}{\sin\lambda} = \frac{(\omega^2R\cos\lambda)}{\sin\gamma}$$

Sínus-regla

(8)

$$\rightarrow \gamma = \arcsin\left\{\frac{(\omega^2R\cos\lambda)\sin\lambda}{|g|}\right\}$$

$$h = r\sin\gamma$$

$$\rightarrow h = r\left\{\frac{\omega^2R\cos\lambda\cdot\sin\lambda}{\sqrt{(\omega^2R\cos\lambda)^2 + g_0^2 - 2g_0\omega^2R\cos\lambda}}\right\} \sim \frac{r\omega^2R\cos\lambda\sin\lambda}{g_0}$$

geisli vatns

$r = 162$ km

$\omega = 7.3\cdot10^{-5}$ rad/s

$\lambda = 47° \sim 0{,}82$ rad

$R = 6.4\cdot10^{6}$ m

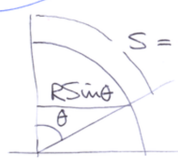
$g_0 = 9.81$ m/s²

Ég fæ $h \sim 7{,}1$ m

(6) Dæmi 10-11 í bók   Loftmótstöðu sleppt   (9)

Ögn send frá N-skauti (norðr yfirb. jarðar) hvar geigun? $T = 10\,\text{min}$, $S = 4800\,\text{km}$

Byrjar á N-skauti, sem er fast í fasta hnitakerfinu
-> notum það. Jörðin snýst meðan á því stendur

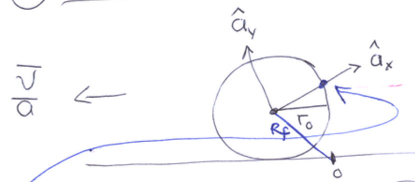


$$\Delta L = \omega_y R\sin\theta \cdot T = \omega_y R \sin\left(\frac{S}{R}\right)\frac{S}{v}$$

$$\Delta\phi = \left(\frac{\Delta L}{2\pi R\sin\theta}\right)2\pi = \omega_y \cdot T = \omega_y \cdot \frac{S}{v}$$

$\Delta L \approx 191\,\text{km}$

(1) Dæmi 10-02 í bók   (10)



Finna punktinn á dekkinu með mesta hröðun m.v. jörð, hver er hröðunin?

Snúningshnitakerfi með miðju í miðju hjóls, þar er hjólið kyrrt, sleppum þyngdarhröðun. Fyrir punkt á dekkinu m.v. jörð gildir (10.23)

$$\bar{a}_f = \ddot{\bar{R}}_f + \bar{a}_r + \dot{\bar{\omega}}\times\bar{r} + \bar{\omega}\times(\bar{\omega}\times\bar{r}) + 2\bar{\omega}\times\bar{v}_r$$

Í snúningshnitakerfinu notum við

$$\ddot{\bar{R}}_f = -a\cos\theta\cdot\hat{e}_x + a\sin\theta\,\hat{e}_y$$

$\bar{r} = r_0\hat{e}_x$ staðsetning punkts á dekki

$\bar{v}_r = 0$ $\bar{\omega} = \frac{v}{r_0}\hat{e}_z$

$\bar{a}_r = 0$ $\dot{\bar{\omega}} = \frac{a}{r_0}\hat{e}_z$

$$2\bar{\omega}\times\bar{v}_r = 0$$

$$\bar{\omega}\times(\bar{\omega}\times\bar{r}) = \left(\frac{v}{r_0}\right)^2 r_0 \cdot \hat{e}_z\times(\hat{e}_z\times\hat{e}_x) = \frac{v^2}{r_0}\hat{e}_z\times\hat{e}_y = -\frac{v^2}{r_0}\hat{e}_x$$

$$\dot{\bar{\omega}}\times\bar{r} = a\,\hat{e}_z\times\hat{e}_x = a\hat{e}_y$$

(11)

$$\to \bar{a}_f = -a\cos\theta\cdot\hat{e}_x + a\sin\theta\cdot\hat{e}_y + a\hat{e}_y - \frac{v^2}{r_0}\hat{e}_x$$

$$= \hat{e}_x\left\{-a\cos\theta - \frac{v^2}{r_0}\right\} + \hat{e}_y\left\{a\sin\theta + a\right\}$$

$$\to \bar{a}_f\cdot\bar{a}_f = |a_f|^2 = \left(a\cos\theta + \frac{v^2}{r_0}\right)^2 + a^2(\sin\theta+1)^2$$

$$= \cancel{a^2\cos^2\theta} + 2a\cos\theta\cdot\frac{v^2}{r_0} + \left(\frac{v^2}{r_0}\right)^2 + \cancel{a^2\sin^2\theta} + a^2 + a^2 2\sin\theta$$

$$= \frac{v^4}{r_0^2} + 2a^2 + \frac{2av^2}{r_0}\cos\theta + a^2 2\sin\theta$$

finnum lággildi

$$\frac{d|a_f|^2}{d\theta} = -\frac{2av^2}{r_0}\sin\theta + a^2 2\cos\theta = 0$$

ef $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{ar_0}{v^2}$

(12)

$$\tan\theta = \frac{ar_0}{v^2}$$

$$\to \cos\theta = \frac{v^2}{\sqrt{(ar_0)^2+v^4}}, \quad \sin\theta = \frac{ar_0}{\sqrt{(ar_0)^2+v^4}}$$

{og auðvitað $\cos^2\theta + \sin^2\theta = 1$}

$$\to \bar{a}_f = -\hat{e}_x\left\{\frac{av^2}{\sqrt{(ar_0)^2+v^4}} + \frac{v^2}{r_0}\right\} + \hat{e}_y a\left\{\frac{ar_0}{\sqrt{(ar_0)^2+v^4}} + 1\right\}$$

$$\to |a_f|^2 = \frac{v^4}{r_0^2} + 2a^2 + \frac{2av^2}{r_0}\frac{v^2}{\sqrt{(ar_0)^2+v^4}} + a^2 2\frac{ar_0}{\sqrt{(ar_0)^2+v^4}}$$

$$= \frac{v^4}{r_0^2} + 2a^2 + \frac{\frac{2av^4}{r_0} + \frac{2a}{r_0}(ar_0)^2}{\sqrt{(ar_0)^2+v^4}}$$

$$\rightarrow |a_f|^2 = \frac{v^4}{r_0^2} + 2a^2 + \frac{2a}{r_0} \frac{v^4 + (ar_0)^2}{\sqrt{(ar_0)^2 + v^4}}$$

$$= \frac{v^4}{r_0^2} + 2a^2 + \frac{2a}{r_0} \sqrt{v^4 + (ar_0)^2}$$

$$= \frac{v^4}{r_0^2} + 2a^2 + 2a\sqrt{\frac{v^4}{r_0^2} + a^2}$$

$$= \left(a + \sqrt{\frac{v^4}{r_0^2} + a^2}\right)^2$$

$$\rightarrow |a_f| = a + \sqrt{\frac{v^4}{r_0^2} + a}$$