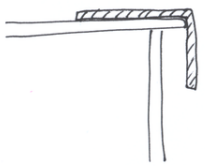
(1) Dæmi 9-21 í bók



L : heildar lengd 1 m

$x(0) = 30\text{cm} = x_0$

$v(0) = 0$

finna τ þegar endi reipis fer yfir brúnina á borðinu.

Massi reipis: m

massi reipis framyfir brún: $\frac{x}{L} \cdot m$

$\rightarrow m\ddot{x} = m\frac{x}{L} \cdot g \rightarrow$ hreyfijafna: $\ddot{x} - \frac{x}{L}g = 0$

Lausn er á $x = Ae^{\omega t} + Be^{-\omega t}$

$\rightarrow \ddot{x} = \omega^2 x$ | $\omega^2 = \frac{g}{L} \rightarrow \omega = \sqrt{\frac{g}{L}}$

finnum A og B til að uppfylla upphafsskilyrði

$x(0) = x_0 = A + B$

$\dot{x}(0) = 0 = A\omega - B\omega$ $\Big\} \rightarrow A = \frac{x_0}{2}$, $B = \frac{x_0}{2}$

$\rightarrow x(t) = x_0 \cosh(\omega t)$

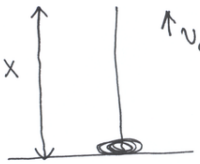
$L = x_0 \cosh(\omega\tau) \rightarrow \frac{L}{x_0} = \cosh(\omega\tau)$

Þá $\tau = \frac{1}{\omega} \operatorname{Arcosh}\left(\frac{L}{x_0}\right) = \sqrt{\frac{L}{g}} \operatorname{Arcosh}\left(\frac{L}{x_0}\right)$

$\tau = \sqrt{\frac{1.0}{9.81}} \operatorname{Arcosh}\left(\frac{1.0}{0.3}\right) \approx 0.6\ \text{s}$

(2) Dæmi 9-44 í bók

Reipi með lengd b og massaþéttleika μ liggur vafið upp á borði. Enda er lítt með hraða v_0 finna kraft á hönd þegar a af reipi er á lofti, $a < b$.



þyngdar kraftur: $F_g = (\mu x)g$

$F_{imp} = \frac{d}{dt}\{(\mu x)v\} = (\mu \frac{dx}{dt})v + \mu x \frac{dv}{dt}$ ($\frac{dv}{dt} = 0$)

$= \mu v v = \mu v_0^2$ (atlagið)

$\rightarrow$ heildar kraftur $F(x) = \mu v_0^2 + \mu x g$

$\rightarrow F(a) = \mu\{v_0^2 + ag\} = \mu ag\{1 + \frac{v_0^2}{ag}\}$

vegna þyngdar | viðbótin vegna atlagsins

(3) Dæmi 9-41 í bók

Gúmmíbolti fellur úr h_1, kemur aftur upp í hæð h_2

finna ϵ (coeff. of restitution) $\epsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|}$ ← eftir / ← fyrir

fallhreyfing $y = v_0 t - \frac{1}{2}gt^2$ og $v = v_0 - gt$

Fyrir árekstur $u_1(0) = 0$, h_1 gefin

$u_1 = -gt_1$ og $h_1 = \frac{1}{2}gt_1^2 \rightarrow t_1 = \sqrt{\frac{2h_1}{g}}$

$\rightarrow u_1 = -g\sqrt{\frac{2h_1}{g}} = -\sqrt{2h_1 g}$

Eftir árekstur (lokatími)

$v_1(t_2) = 0 = v_1(0) - gt_2 \rightarrow t_2 = \frac{v_1(0)}{g}$

$\rightarrow h_2 = v_1(0)t_2 - \frac{1}{2}gt_2^2 = \frac{(v_1(0))^2}{g} - \frac{1}{2}g\frac{(v_1(0))^2}{g^2}$

$= \frac{(v_1(0))^2}{2g} \rightarrow v_1(0) = \sqrt{2h_2 g}$

$$\rightarrow \epsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|} = \frac{\sqrt{2h_2 g}}{\sqrt{2h_1 g}} = \sqrt{\frac{h_2}{h_1}}$$

Hreyfiorka tapað $\quad \Delta T = T_i - T_f = \left(u_1^2 - (v_1(0))^2\right) m$

$$T_i = m u_1^2$$

$$\Rightarrow \frac{\Delta T}{T_i} = \frac{u_1^2 - (v_1(0))^2}{u_1^2} = 1 - \left(\frac{(v_1(0))^2}{u_1^2}\right) = 1 - \frac{h_2}{h_1}$$

$$= 1 - \epsilon^2$$

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(4) Dæmi 9-06 í bók $\quad \bar{r}_1(0) = 0, \ \bar{r}_2(0) = 0$

Tveir massar m_1 og m_2, með $m_2 = m_1 = m$

$\bar{F}_1 = 0$ og $\bar{F}_2 = F_0 \hat{e}_x$, $\bar{v}_1(0) = 0$, $\bar{v}_2(0) = 0$

Finna staðsetn, hraða og hröðun CM

Engin kraftur á $m_2 \rightarrow \bar{r}_2(t) = 0$, $\bar{v}_2(0) = 0$

$$\bar{F}_2 = F_0 \hat{e}_x \rightarrow m\ddot{\bar{r}}_2 = F_0 \hat{e}_x \rightarrow \bar{r}_2(t) = \frac{F_0}{2m} t^2 \cdot \hat{e}_x$$

$$\bar{r}_{CM}(t) = \frac{m_1 \bar{r}_1(t) + m_2 \bar{r}_2(t)}{m_1 + m_2} = \frac{F_0}{4m} t^2 \cdot \hat{e}_x$$

$$\rightarrow \bar{v}_{CM} = \frac{F_0}{2m} t \cdot \hat{e}_x \rightarrow \bar{a}_{CM} = \frac{F_0}{2m} \hat{e}_x$$

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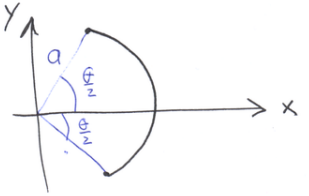
(5) Dæmi 9-04 í bók



finna CM

samhverfa gefur $y_{CM} = 0$

$$X_{CM} = \frac{1}{M} \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} x \, dm \qquad M = s\rho = (a\theta)\cdot\rho$$

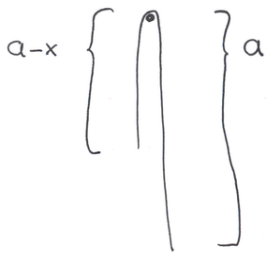
$$dm = a\rho d\theta$$

$$= \frac{1}{M} \int_{-\theta/2}^{\theta/2} d\theta' \cdot a\rho \cdot x \qquad x = a\cos\theta$$

$$= \frac{1}{M} \int_{-\theta/2}^{\theta/2} d\theta' \cdot a^2\rho \cos\theta' = \frac{a^2\rho}{a\theta\rho} \int_{-\theta/2}^{\theta/2} dx \cdot \cos x$$

$$= \frac{a}{\theta} \left\{ \sin x \Big|_{-\theta/2}^{\theta/2} \right\} = \frac{a}{\theta} 2\sin\left(\frac{\theta}{2}\right) = X_{CM}$$

(6) Dæmi 9-20 í bók $\quad$ reipi á vagla, lengd $2a$



$$F_g = \{(a+x)\rho\} g - \{(a-x)\rho\} g$$

$$= (2x\rho) g$$

Finna hröðun þegar reipið fer af vaglinum

Hér er m fasti

$$\frac{dp}{dt} = F_g$$

$$(2a\rho)\dot{v} = (2x\rho)g \rightarrow a\dot{v} = xg$$

Vilyum hraða sem fall af x en ekki t $\rightarrow \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

$$\rightarrow a v \frac{dv}{dx} = xg$$

$$\rightarrow v\,dv = \frac{xg}{a} dx$$

← på er $a+x = 2a$ og reipet for fram at

$$\int_0^v v' dv' = \frac{g}{a}\int_0^a x\,dx \quad \rightarrow \quad \frac{v^2}{2} = \frac{g}{a}\frac{a^2}{2} = \frac{g}{2}a$$

$$\rightarrow \quad v = \sqrt{ga}$$
