(1)

Hreyfijafna

$$m\dot{\bar{v}} = \bar{F}_{ext} + \bar{F}_{rad}\,, \qquad \bar{F}_{rad} = m\tau\ddot{\bar{v}}$$

$\rightarrow$ hreyfijafna kerfisins er

$$m\left\{\dot{\bar{v}} - \tau\ddot{\bar{v}}\right\} = \bar{F}_{ext}$$

þegar $\bar{F}_{ext} = 0$ fæst

$$\dot{\bar{v}} - \tau\ddot{\bar{v}} = 0 \quad \rightarrow \quad \ddot{\bar{v}} = \frac{1}{\tau}\dot{\bar{v}}$$

Umskrifum

$$\dot{\bar{a}} = \frac{1}{\tau}\bar{a} \quad \rightarrow \quad \frac{d\bar{a}}{dt} = \frac{1}{\tau}\bar{a}$$

$$\rightarrow \quad \frac{da}{a} = \frac{1}{\tau}dt \quad \rightarrow \quad \int_{a_0}^{a}\frac{da'}{a'} = \frac{1}{\tau}\int_0^t dt'$$

(2)

$$\ln\left[\frac{a}{a_0}\right] = \frac{1}{\tau}t \quad \rightarrow \quad \bar{a}(t) = \bar{a}_0 e^{t/\tau}$$

Ef $\bar{a}_0 \neq 0$ þá er þetta „run away solution" sem er ekki eðlisfræðileg. Því verðum við að segja að $\bar{F}_{ext} = 0$ gefi okkur að eindinni sé ekki hraðað

$\rightarrow$ engin geislun.

Abraham-Lorentz jafnan er óvenjuleg. Frekari lausn á henni leiðir til forhröðunar sem verður á tímanum $-\tau \rightarrow 0$ sem uppfyllir ekki orsakar lögmálið. Fyrir rafeindir $\tau \sim 10^{-23}$ s þar búumst við ekki við góðri klassískri lýsingu.....

(3)

Dæmi 4-4 í bók

Lord Rayleigh

$$\ddot{x} - (a - b\dot{x}^2)\dot{x} + \omega_0^2 x = 0$$

Athugum breytuskiptin $y = y_0\sqrt{\frac{3b}{a}}\,\dot{x}$ eftir tímaafleiðu

Tímaafleiða

$$\dddot{x} - (a - b\dot{x}^2)\ddot{x} + 2b\dot{x}\ddot{x}\dot{x} + \omega_0^2\dot{x} = 0$$

$$\rightarrow \quad \dddot{x} - (a - 3b\dot{x}^2)\ddot{x} + \omega_0^2\dot{x} = 0$$

Innsetning

$$y = y_0\sqrt{\frac{3b}{a}}\,\dot{x} \quad \rightarrow \quad \dot{y} = y_0\sqrt{\frac{3b}{a}}\,\ddot{x}\,, \quad \ddot{y} = y_0\sqrt{\frac{3b}{a}}\,\dddot{x}$$

$$\rightarrow \quad \dot{x} = \sqrt{\frac{a}{3b}}\,\frac{y}{y_0}\,, \quad \ddot{x} = \sqrt{\frac{a}{3b}}\,\frac{\dot{y}}{y_0}\,, \quad \dddot{x} = \sqrt{\frac{a}{3b}}\,\frac{\ddot{y}}{y_0}$$

(4)

Setjum inn í jöfnu

$$\frac{1}{y_0}\sqrt{\frac{a}{3b}}\left\{\ddot{y} - \left[a - a\frac{y^2}{y_0^2}\right]\dot{y} + \omega_0^2 y\right\} = 0$$

sem verður

$$\ddot{y} - a\left(1 - \frac{y^2}{y_0^2}\right)\dot{y} + \omega_0^2 y = 0$$

$$\ddot{y} + \frac{a}{y_0^2}\left(y^2 - y_0^2\right)\dot{y} + \omega_0^2 y = 0$$

Jafna van der Pols var

$$\ddot{x} + \mu\left(x^2 - a^2\right)\dot{x} + \omega_0^2 x = 0$$

$\rightarrow$ í okkar jöfnu er

$$\frac{a}{y_0^2} = \mu \quad \text{og} \quad y_0^2 = a^2$$

③ Jafna van der Pols

$$\ddot x + \mu(x^2-a^2)\dot x + \omega_0^2 x = 0$$

Setjum sem svo að x og a séu víddarlausar stærðir
μ getur ekki verið það

$$[\omega_0] = \frac{1}{T} \quad \text{og} \quad [\mu] = \frac{1}{T}$$

því er aðeins hægt að ræða hvað gerist þegar $\frac{\mu}{\omega_0}$ er smá stærð, endurritum

$$\ddot x + \omega_0^2 x = -\frac{\mu}{\omega_0}(x^2-a^2)\omega_0\dot x = \frac{\mu}{\omega_0}(a^2-x^2)\omega_0\dot x$$

Ótruflaða lausnin þegar $\mu=0$ gæti verið

$$x(t) = b\cos(\omega_0 t)$$

hreintóna sveifill
með tíðni ω_0

Gerum ráð fyrir að lausnin fyrir smátt μ/ω_0 sé

$$x(t) = \underbrace{b\cos(\omega_0 t)} + u(t)$$

Athugum jöfnu fyrir u:

$$0 + \ddot u + \omega_0^2 u = \frac{\mu}{\omega_0}\left\{a^2 - \left[b\cos(\omega_0 t)+u\right]^2\right\}\omega_0\left\{-b\omega_0\sin(\omega_0 t) + \dot u\right\}$$

$$\to \ddot u + \omega_0^2 u = \frac{\mu}{\omega_0}\left\{a^2 - b^2\cos^2(\omega_0 t) - 2bu\cos(\omega_0 t) - u^2\right\} \cdot \omega_0\left\{-b\omega_0\sin(\omega_0 t)+\dot u\right\}$$

u er í réttu hlutfalli við $\frac{\mu}{\omega_0}$, sleppum því liðum með u-i hægra megin
að minnsta kosti

$$\ddot u + \omega_0^2 u = -\frac{\mu}{\omega_0}\left\{\omega_0\left(a^2 - b^2\cos^2(\omega_0 t)\right) b\omega_0\sin(\omega_0 t)\right\}$$

$$= -\frac{\mu}{\omega_0}\left\{\omega_0^2\left[a^2 b\sin(\omega_0 t) - b^3\cos^2(\omega_0 t)\sin(\omega_0 t)\right]\right\}$$

notum

$$\cos^2(\omega_0 t)\sin(\omega_0 t) = \frac{1}{3}\left\{\sin(3\omega_0 t) + \sin^3(\omega_0 t)\right\}$$

$$= \frac{1}{3}\left\{\sin(3\omega_0 t) + \frac{3\sin(\omega_0 t) - \sin(3\omega_0 t)}{4}\right\}$$

$$= \frac{1}{4}\sin(3\omega_0 t) + \frac{1}{4}\sin(\omega_0 t)$$

því fæst

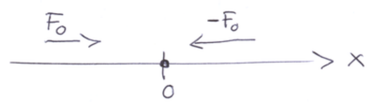
$$\ddot u + \omega_0^2 u = -\frac{\mu}{\omega_0}\omega_0^2 b\left\{a^2\sin(\omega_0 t) - \frac{b^2}{4}\sin(\omega_0 t) - \frac{b^2}{4}\sin(3\omega_0 t)\right\}$$

$$= -\frac{\mu}{\omega_0}\omega_0^2 b\left\{\left(a^2 - \frac{b^2}{4}\right)\sin(\omega_0 t) - \frac{b^2}{4}\sin(3\omega_0 t)\right\}$$

því sést að í línulegri nálgun fyrir $\frac{\mu}{\omega_0}$ er jafnan eins og jafna hreintóna sveifils með tíðni ω_0, en í henni eru þvingunarliðir með tíðni ω_0 og $3\omega_0$

④ dæmi 4-8 í bók

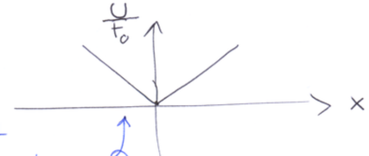
1.D-hreyfing



Engin deyfing

$$F = -\frac{\partial}{\partial x}U(x) \qquad F = \begin{cases} F_0 & \text{ef } x<0 \\ -F_0 & \text{ef } x>0 \end{cases}$$

$$\to U(x) = \begin{cases} -F_0 x & \text{ef } x<0 \\ +F_0 x & \text{ef } x>0 \end{cases} \qquad \to U(x) = F_0|x|$$



Hver grein gæti verið
mætti í föstu þyngdarsviði

Veljum $x(0) = A$, $\dot{x}(0) = 0$, mesta útslag

Hreyfijafnan er ekki heppileg vegna brotsins, en

$$E = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + F_0|x| \quad , \quad |x| \leq A$$

upphafsgildin gefa að $\qquad E = F_0 A$

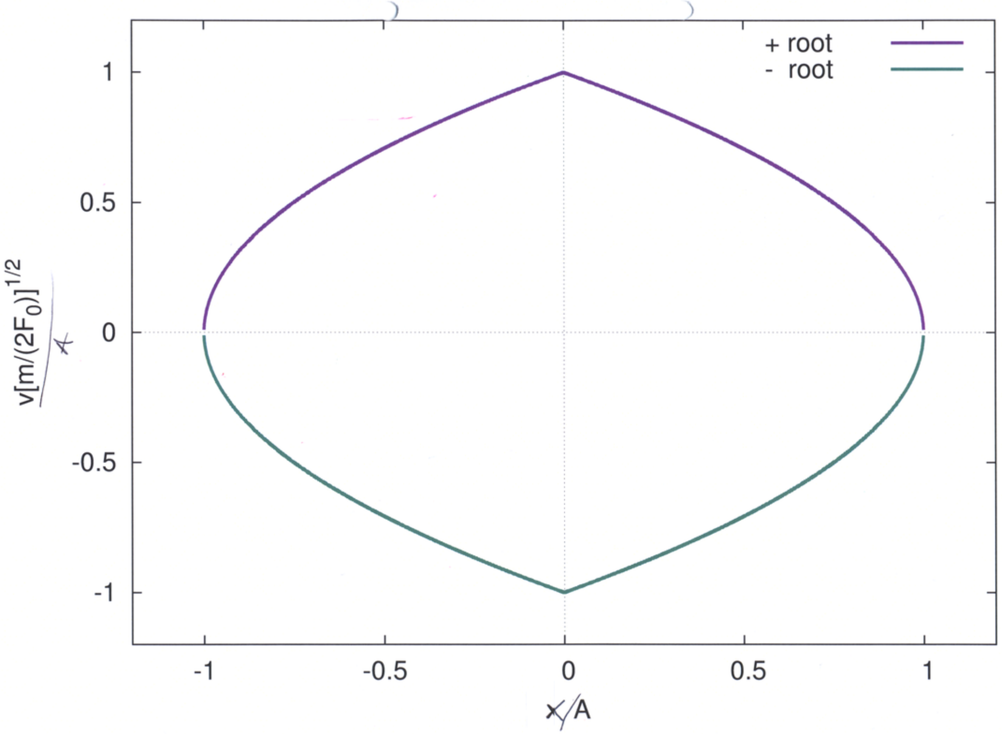
$$\rightarrow \frac{1}{2}mv^2 + F_0|x| = F_0 A \rightarrow v^2 = \frac{2F_0}{m}(A - |x|)$$

$$\rightarrow v = \pm\sqrt{\frac{2F_0}{m}(A-|x|)}$$

fyrir graf

$$v = \pm A\sqrt{\frac{2F_0}{m}}\sqrt{\left(1 - \frac{|x|}{A}\right)} \rightarrow \frac{\dot{x}(x)}{A}\cdot\sqrt{\frac{m}{2F_0}} = \sqrt{1 - \frac{|x|}{A}}$$

Sjá næstu síðu, þar sem fasarit er sýnt



Finnum lotuna

sveiflan er samhverf

skoðum hreyfijöfnuna fyrir $x > 0$

$$m\ddot{x} = -F_0$$

finnum $\tau/4$ þegar sveifillinn fer frá $x = A$ í $x = 0$

$$\ddot{x} = \frac{d^2x}{dt^2} = -\frac{F_0}{m} = \frac{dv}{dt}$$

$$\rightarrow \int_0^v dv' = -\frac{F_0}{m}\int_0^t dt' \rightarrow v(t) = -\frac{F_0}{m}t = \frac{dx}{dt}$$

$$\rightarrow \int_A^0 dx = -\frac{F_0}{m}\int_0^{\tau/4} t'dt' \rightarrow A = \frac{F_0\tau^2}{2m16} \rightarrow \tau^2 = \frac{32mA}{F_0}$$

$$\rightarrow \tau = 4\sqrt{\frac{2mA}{F_0}}$$

(5) Þvingaði deyfði sveifillinn ~~eftir~~ svipuðu lausnina um hann gildir

$$\dot{x} = \frac{-A\omega}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\omega^2\beta^2}}\sin(\omega t - \delta)$$

samanber jöfnu (3.60) í bók

því er „hraða útslagið"

$$v_{max} = \frac{A\omega}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\omega^2\beta^2}}$$

Við viljum vita fyrir hvaða $\omega = \omega_v$ fæst max gildi

$$\left.\frac{\partial v_{max}}{\partial \omega}\right|_{\omega=\omega_v} = 0$$

$$\frac{\partial V_{max}}{\partial\omega} = \frac{A}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}} - \frac{\frac{1}{2}A\omega\{4\omega^3-4\omega_0^2\omega+8\beta^2\omega\}}{[(\omega_0^2-\omega^2)^2+4\omega^2\beta^2]^{3/2}} = 0$$

$$\to \quad \{\omega_0^4+\omega^4-\cancel{2\omega_0^2\omega^2}+\cancel{4\omega^2\beta^2}-2\omega^4+\cancel{2\omega_0^2\omega^2}-\cancel{4\beta^2\omega^2}\} = 0$$

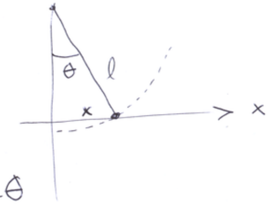
$$\to \quad \omega_0^4-\omega^4 = 0 \qquad \to \quad \omega = \omega_0$$

Þetta öllu heldur $\underline{\omega_v = \omega_0}$ er $\underline{\text{hermutíðni hraða}}$

Hermutíðnin fyrir útslagið var $\underline{\omega_R^2 = \omega_0^2 - 2\beta^2}$

6) Dæmi 4-7 í bók

pendúll $\boxed{\ddot\theta + \omega_0^2\sin\theta = 0}$ $\omega_0^2 = \frac{g}{l}$



skoðum lárétta hreyfingu $x = l\sin\theta$

$$\sin\theta = \frac{x}{l} \quad \to \quad \theta = \arcsin\left(\frac{x}{l}\right)$$

$$\to \quad \dot\theta = \frac{1}{\sqrt{1-\left(\frac{x}{l}\right)^2}}\frac{\dot x}{l} \qquad \to \quad \ddot\theta = \frac{\ddot x}{l}\frac{1}{\sqrt{1-\left(\frac{x}{l}\right)^2}} + \frac{\frac{x}{l}\frac{\dot x}{l}\frac{\dot x}{l}}{\left(1-\left(\frac{x}{l}\right)^2\right)^{3/2}}$$

$\left(\frac{\dot x}{l}\right)^2$ minnir á orku — heildarorka — max stöðuorka

$$\frac{1}{2}ml^2\dot\theta^2 - mgl\cos\theta = E = -mgl\cos\theta_0$$

$$\frac{1}{2}\dot\theta^2 - \omega_0^2\cos\theta = -\omega_0^2\cos\theta_0$$

$$\to \quad \dot\theta^2 = 2\omega_0^2\{\cos\theta - \cos\theta_0\}$$

endurritum sem

$$\frac{1}{1-\left(\frac{x}{l}\right)^2}\left(\frac{\dot x}{l}\right)^2 = 2\omega_0^2\left\{\sqrt{1-\left(\frac{x}{l}\right)^2} - \sqrt{1-\left(\frac{x_0}{l}\right)^2}\right\}$$

Setjum saman

$$\ddot\theta + \omega_0^2\sin\theta = 0$$

$$\frac{\ddot x}{l}\frac{1}{\sqrt{1-\left(\frac{x}{l}\right)^2}} + \frac{1}{\sqrt{1-\left(\frac{x}{l}\right)^2}}\frac{x}{l}\left\{\sqrt{1-\left(\frac{x}{l}\right)^2} - \sqrt{1-\left(\frac{x_0}{l}\right)^2}\right\}2\omega_0^2 + \omega_0^2\frac{x}{l} = 0$$

viljum halda liðum upp í $(x/l)^3$

$$\frac{\ddot x}{l} + \frac{x}{l}\left\{\sqrt{1-\left(\frac{x}{l}\right)^2} - \sqrt{1-\left(\frac{x_0}{l}\right)^2}\right\}2\omega_0^2 + \omega_0^2\frac{x}{l}\sqrt{1-\left(\frac{x}{l}\right)^2} = 0$$

Notum $\sqrt{1-\left(\frac{x}{l}\right)^2} \approx 1 - \frac{1}{2}\left(\frac{x}{l}\right)^2 - \frac{1}{8}\left(\frac{x}{l}\right)^4 + \dots\dots$

$$\frac{\ddot x}{l} + \frac{x}{l}\left\{-\frac{1}{2}\left(\frac{x}{l}\right)^2 + \frac{1}{2}\left(\frac{x_0}{l}\right)^2\right\}2\omega_0^2 + \omega_0^2\frac{x}{l}\left\{1 - \frac{1}{2}\left(\frac{x}{l}\right)^2\right\} = 0$$

$$\to \quad \boxed{\frac{\ddot x}{l} + \omega_0^2\left\{1 + \left(\frac{x_0}{l}\right)^2\right\}\frac{x}{l} - \frac{3}{2}\omega_0^2\left(\frac{x}{l}\right)^3 = 0}$$

Í bókinni er skrifað eftir margföldun með l

$$\ddot x + \omega_0^2\left\{1 - \left(\frac{x_0}{l}\right)^2\right\}x - \frac{3}{2}\frac{g}{l^3}x^3 = 0$$