(1) 3-13 í bók, Markdeyfður sveifill (1)

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \quad \to \omega_0^2 = \beta^2$$

$$\ddot{x} + 2\beta\dot{x} + \beta^2 x = 0 \quad \leftarrow \text{hreyfij.}$$

Reynum lausn á ham $x(t) = y(t)e^{-\beta t}$

$$\to \dot{x} = \dot{y}e^{-\beta t} - \beta y e^{-\beta t} = e^{-\beta t}\{\dot{y} - \beta y\}$$

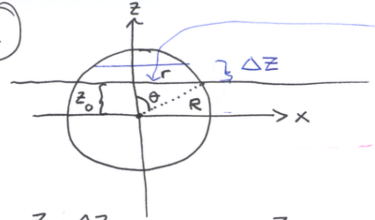
$$\ddot{x} = -\beta e^{-\beta t}\{\dot{y} - \beta y\} + e^{-\beta t}\{\ddot{y} - \beta\dot{y}\} = e^{-\beta t}\{\ddot{y} - 2\beta\dot{y} + \beta^2 y\}$$

Setjum inn í hreyfij.

$$e^{-\beta t}\{\ddot{y} - 2\beta\dot{y} + \beta^2 y + 2\beta\dot{y} - 2\beta^2 y + \beta^2 y\} = 0$$

$\to \ddot{y} = 0$ með lausn $y = A + Bt$ $\to x(t) = \{A + Bt\}e^{-\beta t}$

(2)



Reiknum rúmmál sneiðar (2)

$r = R\sin\theta$, $\quad z_0 = R\cos\theta_0$

$r^2 = R^2 - z^2$

$$\int_{z_0}^{z_0+\Delta z} \pi r^2 dz' = \int_{z_0}^{z} \pi(R^2 - z'^2)dz' = \pi\left(R^2 z' - \frac{z'^3}{3}\right)\Bigg|_{z_0}^{z} = V(z, z_0)$$

$$= \pi\left\{R^2(z - z_0) - \frac{z^3 - z_0^3}{3}\right\}$$

Hér er gott að sannfæra sig um að $V(R,0)$ er $\frac{2\pi R^3}{3}$ ← hálf kúla

Á kúluna verkar þyngdarkraftur $-Mg = -\rho V_0 g$ og flotkraftur þegar kúlan marar með miðju z_0 undir yfirborði eru þeir í jafnvægi

Þegar ég ýti henni niður um Δz verkar flotkraftur vegna $V(z_0 + \Delta z, z_0)$ beint upp (3)

$$F_f(\Delta z) = \rho_0 V(z_0 + \Delta z, z_0) g = \rho_0 g\pi\left\{R^2\Delta z - \frac{(z_0 + \Delta z)^3 - z_0^3}{3}\right\}$$

$$= \rho_0 g\pi\left\{\Delta z(R^2 - z_0^2) - (\Delta z)^2 z_0 - (\Delta z)^3\right\}$$

$$Ma = \rho V_0(\ddot{\Delta z}) = -\rho_0 g\pi\left\{\Delta z(R^2 - z_0^2) - (\Delta z)^2 z_0 - (\Delta z)^3\right\}$$

$$\to (\ddot{\Delta z}) = -\frac{g\rho_0\pi}{\rho V_0}\left\{\Delta z(R^2 - z_0^2) - (\Delta z)^2 z_0 - (\Delta z)^3\right\}$$

Ef við tökum bara línulega liðinn? (4)

$$(\ddot{\Delta z}) + \left\{\frac{g\rho_0\pi}{\rho V_0}(R^2 - z_0^2)\right\}\Delta z = 0$$

ef hún er í kafi $z_0 = R$ → engin sveifla

sveiflan er háð z_0

Annars er afleiðujafnan ólínuleg

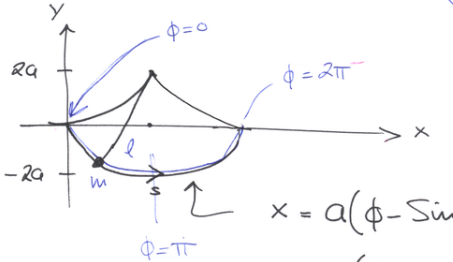
Ef hún er hálf í kafi $z_0 = 0$

$$\to (\ddot{\Delta z}) + \left\{\frac{g\rho_0\pi R^2}{\rho V_0}\right\}\left\{\Delta z - \frac{1}{R^2}(\Delta z)^3\right\}$$

Jafnvel þá er sveiflan ósamhverf

(3) Hjólferils sveifill

(5)



Skilgreining ferils

$$x = a(\phi - \sin\phi) \;\to\; dx = a(1-\cos\phi)\,d\phi \qquad (*)$$

$$y = a(\cos\phi - 1) \;\to\; dy = -a\sin\phi\, d\phi$$

Eftir ferlinum

$$(ds)^2 = (dx)^2 + (dy)^2 = a^2\left\{(1-\cos\phi)^2 + \sin^2\phi\right\}(d\phi)^2$$

$$= 2a^2(1-\cos\phi)(d\phi)^2 = 4a^2\sin^2\left(\frac{\phi}{2}\right)\cdot(d\phi)^2$$

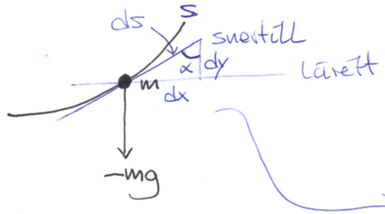
$$\to\; ds = 2a\sin\left(\frac{\phi}{2}\right)\cdot d\phi$$

(6)

$$\to\; v = \frac{ds}{dt} = 2a\sin\left(\frac{\phi}{2}\right)\frac{d\phi}{dt} = -4a\frac{d}{dt}\left\{\cos\left(\frac{\phi}{2}\right)\right\}$$

$$\to\; \dot v = -4a\frac{d^2}{dt^2}\left\{\cos\left(\frac{\phi}{2}\right)\right\}$$

Hvað með kraftinn



$F = -mg\cos\alpha$

þurfum að tengja α og ϕ

$$\frac{dy}{ds} = \cos\alpha \qquad (**)$$

En $(*)$

$$\frac{dy}{ds} = -\frac{a\sin\phi\cdot d\phi}{2a\sin\left(\frac{\phi}{2}\right)d\phi} = -\frac{\sin\phi}{2\sin\left(\frac{\phi}{2}\right)}$$

(7)

Hálfa hornið truflar aðeins

en

$$\left(2\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)\right)^2 = (1-\cos\phi)(1+\cos\phi) = 1-\cos^2\phi = \sin^2\phi.$$

$$\to\; -\frac{\sin\phi}{2\sin\left(\frac{\phi}{2}\right)} = -\cos\left(\frac{\phi}{2}\right) \;\to\; \frac{dy}{ds} = -\cos\left(\frac{\phi}{2}\right)$$

$$= \cos\alpha$$ notum $(**)$

því er $F = +mg\cos\left(\frac{\phi}{2}\right)$, $m\dot v = -4am\frac{d^2}{dt^2}\left\{\cos\left(\frac{\phi}{2}\right)\right\}$

Skiptum um breytu $z = \cos\left(\frac{\phi}{2}\right)$

$$\to\; -4a\ddot z = gz \;\to\; \boxed{\ddot z + \left\{\frac{g}{4a}\right\}z = 0}$$

$\to$ hreintóna sveifill með $\omega_0 = \sqrt{\frac{g}{4a}}$ óháð útslagi

(4) Dæmi 3-11 í bók, deyfður hreintóna sveifill (sambærilegt við fyrirlestur 3 bls. 16)

(8)

$$E = T + U = \frac{1}{2}m\dot x^2 + \frac{1}{2}kx^2 \qquad \omega_1 = \sqrt{\omega_0^2+\beta^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$x = Ae^{-\beta t}\cos(\omega_1 t - \delta)$$

$$\dot x = Ae^{-\beta t}\left\{-\beta\cos(\omega_1 t-\delta) - \omega_1\sin(\omega_1 t-\delta)\right\}$$

$$\to\; E(t) = \frac{A^2}{2}e^{-2\beta t}\Big\{(m\beta^2+k)\cos^2(\omega_1 t-\delta) + m\omega_1^2\sin^2(\omega_1 t-\delta) + 2m\beta\omega_1\sin(\omega_1 t-\delta)\cos(\omega_1 t-\delta)\Big\}$$

Notum

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

og

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\to\; E(t) = \frac{mA^2}{2}e^{-2\beta t}\left\{\beta^2\cos(2(\omega_1 t-\delta)) + \beta\sqrt{\omega_0^2-\beta^2}\,\sin(2(\omega_1 t-\delta)) + \omega_0^2\right\}$$

9

$$\to \dot E(t) = \frac{mA^2}{2} e^{-2\beta t}\Big\{ -2\omega_1\beta^2 \sin(2(\omega_1 t-\delta)) + 2\omega_1\beta\sqrt{\omega_0^2-\beta^2}\cos(2(\omega_1 t-\delta))$$
$$- 2\beta\beta^2\cos(2(\omega_1 t-\delta)) - 2\beta^2\sqrt{\omega_0^2-\beta^2}\sin(2(\omega_1 t-\delta))$$
$$- 2\beta\omega_0^2 \Big\}$$

$$= \frac{mA^2}{2}e^{-2\beta t}\Big\{(2\beta\omega_0^2-4\beta^3)\cos(2(\omega_1 t-\delta)) - 4\beta\sqrt{\omega_0^2-\beta^2}\sin(2(\omega_1 t-\delta))$$
$$-2\beta\omega_0^2\Big\}$$

$$\langle \dot E(t)\rangle_{lota} = -\frac{mA^2}{2}2\beta\omega_0^2 e^{-2\beta t} \quad \text{ef } \beta \ll \omega_0$$
$$= -m\beta\omega_0^2A^2e^{-2\beta t}$$

10

6 Dæmi 3-4 í bók

Hreintóna sveifill

$$x = A\sin(\omega_0 t) \qquad \text{með } \omega_0 = \sqrt{\frac{k}{m}}$$
$$\dot x = \omega_0 A\cos(\omega_0 t)$$

Lotan er $\tau = \frac{2\pi}{\omega_0}$ → tíma meðaltal yfir eina lotu

$$\langle T\rangle_t = \frac{1}{\tau}\int_t^{t+\tau} dt' \frac{m}{2}\dot x^2 = \frac{m}{2\tau}A^2\omega_0^2\underbrace{\int_t^{t+\tau}\cos^2(\omega_0 t')dt'}_{=\frac{\tau}{2}}$$
$$= \frac{mA^2\omega_0^2}{4}$$
$$\langle U\rangle_t = \frac{1}{\tau}\int_t^{t+\tau} dt' \frac{kx^2}{2} = \frac{k}{2\tau}A^2\int_t^{t+\tau}\sin^2(\omega_0 t')dt' = \frac{kA^2}{4} = \frac{mA^2\omega_0^2}{4}$$

11

$$\to \langle T\rangle_t = \langle U\rangle_t$$

$$\langle T\rangle_x = \frac{1}{A}\int_0^A \frac{m}{2}\dot x^2\, dx \ , \quad \langle U\rangle_x = \frac{1}{A}\int_0^A \frac{kx^2}{2}\, dx$$

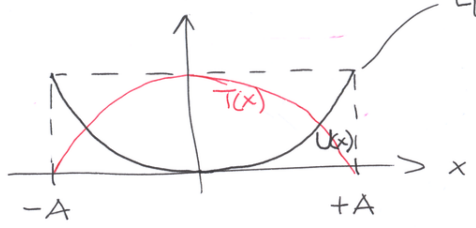
$$\langle U\rangle_x = \frac{m\omega_0^2A^2}{6}$$ og síðan notum

$$\dot x^2 = \omega_0^2A^2\cos^2(\omega_0 t)$$
$$= \omega_0^2A^2\{1-\sin^2(\omega_0 t)\}$$
$$= \omega_0^2(A^2-x^2)$$

$$\to \langle T\rangle_x = 2\frac{m\omega_0^2A^2}{6} = 2\langle U\rangle_x$$

12

Af hverju

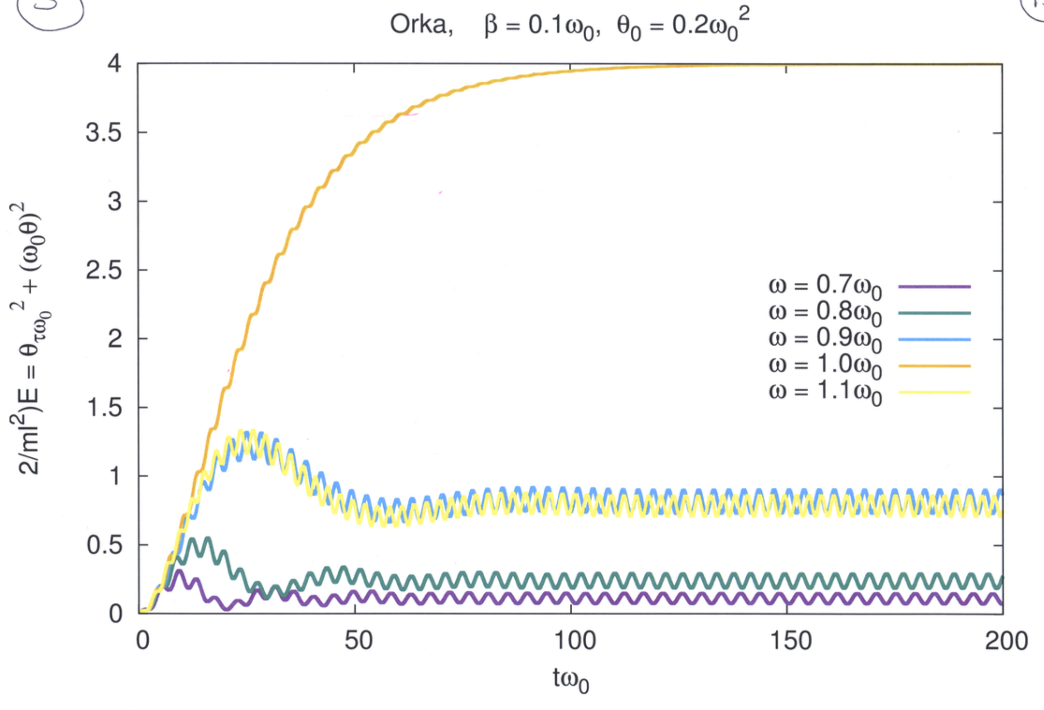


$E_{heild} = \frac{1}{2}mA^2\omega_0^2$ = fasti

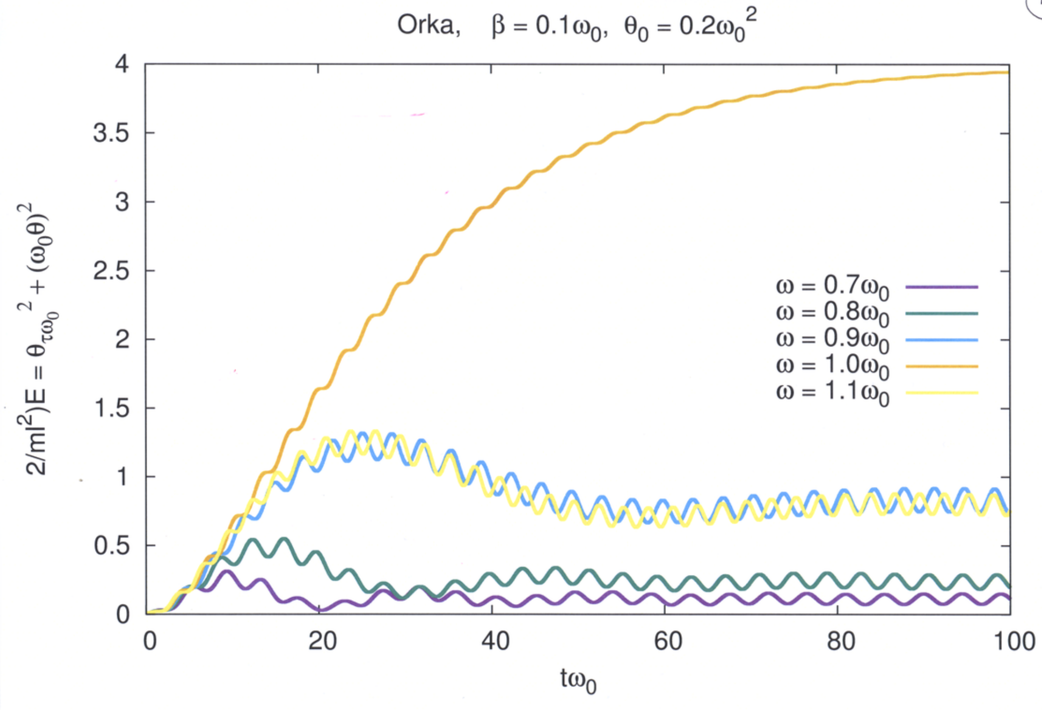
Flatarmálið er mism

5

13


Orka, $\beta = 0.1\omega_0$, $\theta_0 = 0.2\omega_0^2$
$2/ml^2)E = \theta_{\tau\omega_0}^2 + (\omega_0\theta)^2$
$t\omega_0$
$\omega = 0.7\omega_0$
$\omega = 0.8\omega_0$
$\omega = 0.9\omega_0$
$\omega = 1.0\omega_0$
$\omega = 1.1\omega_0$
0
0.5
1
1.5
2
2.5
3
3.5
4
0
50
100
150
200

14


Orka, $\beta = 0.1\omega_0$, $\theta_0 = 0.2\omega_0^2$
$2/ml^2)E = \theta_{\tau\omega_0}^2 + (\omega_0\theta)^2$
$t\omega_0$
$\omega = 0.7\omega_0$
$\omega = 0.8\omega_0$
$\omega = 0.9\omega_0$
$\omega = 1.0\omega_0$
$\omega = 1.1\omega_0$
0
0.5
1
1.5
2
2.5
3
3.5
4
0
20
40
60
80
100