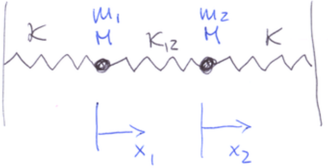
Tveir tengdir sveiflar enn



Höfum séð að

$$\mathbb{A} = \begin{bmatrix} K+K_{12} & -K_{12} \\ -K_{12} & K+K_{12} \end{bmatrix}$$

$$\mathbb{M} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} = M\mathbb{1}$$

Þurfum að leysa

$$\mathbb{A}\bar{a} = \omega^2 \mathbb{M}\bar{a} = \omega^2 M\bar{a}$$

Eigingildin eru

$$\omega_1^2 M = K + 2K_{12} \rightarrow \omega_1 = \sqrt{\frac{K+2K_{12}}{M}}$$

$$\omega_2^2 M = K \rightarrow \omega_2 = \sqrt{\frac{K}{M}}$$

með eiginvigra (óstöðluða)

$$\bar{a}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{og} \quad \bar{a}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

stöðluð ummyndun

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$A\bar{a} = \omega^2 M \bar{a}$$ ekki á hornalínuham

$$U^t A U (U^t \bar{a}) = \omega^2 M (U^t \bar{a})$$ á hornalínuham

Bíðum með stöðlun á U

$$\bar{a} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longrightarrow U^t \bar{a} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_2 + x_1 \end{pmatrix} = \bar{\eta}$$

$$U^t \bar{a} = \bar{\eta} \longrightarrow \bar{a} = U\bar{\eta} = \begin{pmatrix} \eta_1 + \eta_2 \\ \eta_2 - \eta_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

η_r eru eiginlausnir með eigintíðni ω_r

setjum $\eta_r(t) = \beta_r e^{i\omega_r t}$

stöðlun + upphafsgildi-vídd, og fasi $e^{-i\delta_r}$

Því er lausnin

$$\begin{cases} x_1(t) = \eta_1(t) + \eta_2(t) = \beta_1 e^{i\omega_1 t} + \beta_2 e^{i\omega_2 t} \\ x_2(t) = \eta_2(t) - \eta_1(t) = -\beta_1 e^{i\omega_1 t} + \beta_2 e^{i\omega_2 t} \end{cases}$$

sem er hægt að bera saman við almenna lausnar formið

$$q_j(t) = \sum_r a_{jr} \eta_r(t)$$

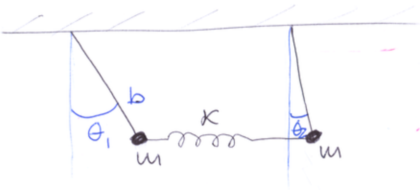
$$\eta_r(t) = \beta_r e^{i\omega_r t}$$

$\eta_1 = 0$ ef $x_1 = x_2$ $\rightarrow$ samfasa sveifla með $\omega_2 = \sqrt{\frac{K}{M}}$

$\eta_2 = 0$ ef $x_1 = -x_2$ $\rightarrow$ andfasa sveifla með $\omega_1 = \sqrt{\frac{K+2K_{12}}{M}}$

í andfasa reynir á K_{12}-gorminn!

Dømi, tveir tengdir pendúlar



$$T = \frac{m}{2}(b\dot{\theta}_1)^2 + \frac{m}{2}(b\dot{\theta}_2)^2$$

$$U = mgb(1-\cos\theta_1) + mgb(1-\cos\theta_2) + \underbrace{\frac{k}{2}\left\{b\sin\theta_1 - b\sin\theta_2\right\}^2}$$

Í javnvági þ.e. $\theta_1 = 0$ og $\theta_2 = 0$ er gormurinn óteygdur og óbjappaður

Gerum ráð fyri lítlum sveiflum, $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2} + \dots$

$$\rightarrow U = \frac{mgb}{2}\left\{\theta_1^2 + \theta_2^2\right\} + \frac{kb^2}{2}(\theta_1 - \theta_2)^2$$

$$\rightarrow M = \begin{bmatrix} mb^2 & 0 \\ 0 & mb^2 \end{bmatrix}, \quad A = \begin{bmatrix} \frac{\partial^2 U}{\partial\theta_1\partial\theta_1} & \frac{\partial^2 U}{\partial\theta_1\partial\theta_2} \\ \frac{\partial^2 U}{\partial\theta_2\partial\theta_1} & \frac{\partial^2 U}{\partial\theta_2\partial\theta_2} \end{bmatrix} = \begin{bmatrix} mgb + kb^2 & -kb^2 \\ -kb^2 & mgb + kb^2 \end{bmatrix}$$

þui fást eigingildin

$$\omega_1^2 mb^2 = mgb + kb^2 \rightarrow \omega_1 = \sqrt{\frac{g}{b} + \frac{2k}{m}}$$

$$\omega_2^2 mb^2 = mgb \rightarrow \omega_2 = \sqrt{\frac{g}{b}}$$

$U \sim \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$U^t \bar{a} = \bar{\eta}$

með eiginvigra

$$\bar{a}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ og } \bar{a}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{aligned} \eta_1 &= \theta_1 - \theta_2 \\ \eta_2 &= \theta_1 + \theta_2 \end{aligned}$$

Þá

$$\theta_1 \sim \eta_1 + \eta_2$$

$$\theta_2 \sim \eta_2 - \eta_1$$

$\eta_1 = 0$ ef $\theta_1 = \theta_2$

$\rightarrow \eta_2$ er virkur með $\omega_2 = \sqrt{\frac{g}{b}}$

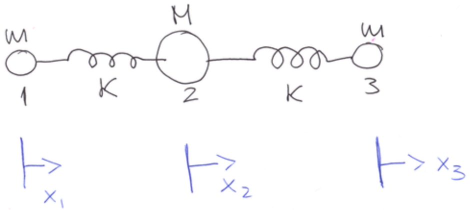
samfasa sveifla þ.a.

K kemur ekki við sögu

$\bar{a} = U\bar{\eta}$

Dæmi

Skoðum línulega sveifluhátti sem geta komið upp í t.d. CO_2



Við viljum sérstaklega sjá hér hvernig mismunandi massar koma inn

Engin hröðun CM $\to$ skorður

$$m\{x_1 + x_3\} + Mx_2 = 0$$

$$\to x_2 = -\frac{m}{M}\{x_1 + x_3\}$$

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_3^2 + \frac{1}{2}M\dot{x}_2^2$$

$$= \frac{m}{2}\left\{\dot{x}_1^2 + \dot{x}_3^2\right\} + \frac{m^2}{2M}\left\{\dot{x}_1^2 + \dot{x}_3^2 + 2\dot{x}_1\dot{x}_3\right\}$$

hraðavíxlverkun

Viljum losna við hroðatengsl,
notum nýj hnit

$$q_1 = x_3 + x_1$$

$$q_2 = x_3 - x_1$$

$$\rightarrow \quad x_3 = \frac{q_1 + q_2}{2}$$

$$x_1 = \frac{q_1 - q_2}{2}$$

$$x_2 = -\frac{m}{M}\left\{x_1 + x_3\right\} = -\frac{m}{M} q_1$$

$$\rightarrow T = \frac{m}{4}\dot{q}_2^2 + \frac{mM + 2m^2}{4M}\dot{q}_1^2$$

$$U = \frac{K}{2}(x_2 - x_1)^2 + \frac{K}{2}(x_3 - x_2)^2$$

$$= \left(\frac{2m + M}{2M}\right)^2 K q_1^2 + \frac{K}{4} q_2^2$$

$$\rightarrow \mathbb{A} = \begin{bmatrix} \frac{K}{2}\left(\frac{2m+M}{M}\right)^2 & 0 \\ 0 & \frac{K}{2} \end{bmatrix}$$

$$\mathbb{M} = \begin{bmatrix} \frac{mM + 2m^2}{2M} & 0 \\ 0 & \frac{m}{2} \end{bmatrix}$$

$A\bar{a} = \omega^2 M \bar{a}$, M er á hornalínuham

$\rightarrow \left(M^{-1}A\right)\bar{a} = \omega^2 \bar{a}$

$$\begin{pmatrix} \frac{2M}{mM + 2m^2} & 0 \\ 0 & \frac{2}{m} \end{pmatrix} \begin{pmatrix} \frac{K}{2}\left(\frac{2m+M}{M}\right)^2 & 0 \\ 0 & \frac{K}{2} \end{pmatrix} = \begin{pmatrix} \frac{K}{mM}(M+2m) & 0 \\ 0 & \frac{K}{m} \end{pmatrix}$$

$\bar{a}$ hornalínuham $\rightarrow$ með eiginvigra

$$\omega_1 = \sqrt{\frac{K}{M} \cdot \frac{M+2m}{m}}$$

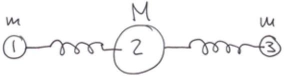
$\bar{a}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ og $\bar{a}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\omega_2 = \sqrt{\frac{K}{m}}$$

$q_1 \sim \eta_1$

$q_2 \sim \eta_2$

$$\left.\begin{aligned} q_1 &= x_3 + x_1 \\ q_2 &= x_3 - x_1 \end{aligned}\right\}$$



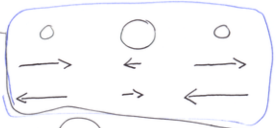
Ef $x_3 - x_1 = 0 \rightarrow \eta_2 = 0 \rightarrow \eta_1$ virkar $\sim q_1$

með ω_1

m_1 og m_3 í fasa

með $x_2 = -\frac{2m}{M} x_1$

úr fasa



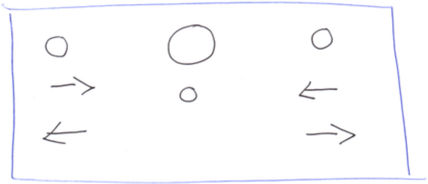
Ef $x_3 + x_1 = 0$

$\eta_1 = 0$ með η_2 virkar

með ω_2

$x_3 = -x_1$ og $x_2 = x_3 + x_1 = 0$

M kyrr með m_1 í andfasa við m_3



Dæmi: 3 pendúlar (margfeldni)

Berum saman við dæmið með 2 tengdum pendúlum

$$T = \frac{Ml^2}{2}\left[\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2\right]$$

$$U = \frac{Mgl + Kl^2}{2}\left[\theta_1^2 + \theta_2^2 + \theta_3^2\right] - Kl^2\left[\theta_1\theta_2 + \theta_1\theta_3 + \theta_2\theta_3\right]$$

$$\mathbb{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} Ml^2 \qquad \mathbb{A} = (Mgl + Kl^2)\begin{bmatrix} 1 & -\epsilon & -\epsilon \\ -\epsilon & 1 & -\epsilon \\ -\epsilon & -\epsilon & 1 \end{bmatrix}$$

þar sem

$$\epsilon = \frac{Kl^2}{Mgl + Kl^2}$$

leysum

$$\mathbb{A}\bar{a} = \omega^2 Ml^2 \bar{a}$$

$$\begin{pmatrix} 1 & -\epsilon & -\epsilon \\ -\epsilon & 1 & -\epsilon \\ -\epsilon & -\epsilon & 1 \end{pmatrix} \bar{a} = \frac{\omega^2 M l^2}{Mgl + Kl^2} \bar{a}$$

$$\to \omega^2 \frac{Ml^2}{Mgl + Kl^2} = \begin{cases} 1 - 2\epsilon & \text{með} \quad \bar{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ 1 + \epsilon & \text{með} \quad \bar{a}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ 1 + \epsilon & \text{með} \quad \bar{a}_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \end{cases}$$

tvöfalt eigingildi

$\bar{a}_1 \cdot \bar{a}_2 = 0$ en $\bar{a}_2 \cdot \bar{a}_3 \neq 0$

$\bar{a}_1 \cdot \bar{a}_3 = 0$

$\bar{a}_3$ og $\bar{a}_2$ spanna hlutrúm og innan þess þarf at mynda tvo hornrætta vigra

Hér er ekki hægt að nota eiginvigrana beint til að mynda hornrétta ummyndun. Þessari ummyndun þarf fyrir alla vigrana

En $\bar{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ má tengja við samfasa sveiflu allra sveiflanna, og hina má tengja við sveiflur úr fasa.

Hér þarf almennari aðferð til að finna hornrétta eiginvigra sem myndað geta hornrétta ummyndun

Í tölulegum reikningum má skekkja kerfið örlítið, og vinna með aðferð Jacobis: sjá James Demmel og Krešimir Veselić, SIAM J. Matrix Anal. & Appl., 13 (4), 1204 (2006)

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Svo tilraunavinna sýnir að við getum haldið

$$\bar{a}_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

og endurnefnd hina þ.a.

$$\bar{a}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix} \quad \text{og} \quad \bar{a}_3 = \frac{1}{\sqrt{6}}\begin{pmatrix}-2\\1\\1\end{pmatrix}$$

Sem verða þá hornréttir, $\bar{a}_2\cdot\bar{a}_3=0$ og eru líka hornréttir á $\bar{a}_1$ $\bar{a}_1\cdot\bar{a}_2=0$ $\bar{a}_1\cdot\bar{a}_3=0$

útbúum

$$U = \begin{pmatrix}\frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\end{pmatrix}$$

og fengið

$$U^t A U = \begin{pmatrix}1-2\epsilon & 0 & 0\\ 0 & 1+\epsilon & 0\\ 0 & 0 & 1+\epsilon\end{pmatrix}$$

Normal hættirnir eru síðan

$$\bar{\eta} = U^t \bar{\theta} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{3}}\theta_1 , & \frac{1}{\sqrt{3}}\theta_2 , & \frac{1}{\sqrt{3}}\theta_3 \\ 0 , & \frac{1}{\sqrt{2}}\theta_2 , & -\frac{1}{\sqrt{2}}\theta_3 \\ -\frac{2}{\sqrt{6}}\theta_1 , & \frac{1}{\sqrt{6}}\theta_2 , & \frac{1}{\sqrt{6}}\theta_3 \end{pmatrix}$$

samfasa sveifla með lægsta tíðnina

tveir í andfasa

tveir í fasa og sá þriðji úr fasa með tvöfalda sveiflu

allir hornréttir

CM-sveifla án innri sveiflu

Engin CM-sveifla bara innri sveifla

Til samanburðar (fyrir nema í eðlisfræði)

Hugsum okkur hreintóna sveifil í skammtafræði

$$H_0 = \hbar\omega\left\{a^+a + \frac{1}{2}\right\}$$

bætum við hann

$$H_I = \hbar\omega f(a^+, a)$$

þ.a. í grunni eiginástanda H_0, $\{|n\rangle\}$ er $H = H_0 + H_I$ ekki hornalínu fylki

Tímaþróun kerfisins er lýst með

$$i\hbar\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$$

Í grunni H_0 útsést þetta sem

sem tengdar afleiðujöfnur

$$i\hbar\partial_t|\alpha(t)\rangle = H|\alpha(t)\rangle$$

þ.s. $|\alpha(t)\rangle$ er ekki með einfalda tímaþróun

Til er ummyndun U þ.a.

$$U^+ H U = H_{diag}$$

þ.s. H_{diag} er á hornalínuham með eigingildi H á hornalínunni

->

$$i\hbar\partial_t\left(U^+|\alpha(t)\rangle\right) = U^+ H U\left(U^+|\alpha(t)\rangle\right)$$

þ.s.

$$|n\rangle e^{-i\omega_n t} = \sum_\alpha C^*_{\alpha n}|\alpha(t)\rangle$$

Sem má skrifa við

$$\sum_n C_{\alpha n} |n) e^{-i\omega_n t} = |\alpha(t))$$

Hér eru

$$H |n) = \hbar\omega_n |n)$$

$$U^{+} H U = \begin{pmatrix} \hbar\omega_0 & & \\ & \ddots & \\ & & \hbar\omega_n \; \ddots \end{pmatrix}$$

$$U^{+} U = \mathbb{1}$$

Hér var ekki „explicit" notað

$$H_0 |n\rangle = E_n^0 |n\rangle \qquad n = 0, 1, \ldots$$

með $E_n^0 = \hbar\omega\left(n + \frac{1}{2}\right)$

Við gáfum okkur engar upplýsingar um orkuróf H, $\hbar\omega_n$