Fallhreyfing við krafti Coriolis – Dømi

Við yfirborð jørðar (kerfi sum snýst) høvdum vit leitt út

allir ytri kreftir

$$\vec{F}_{eff} = \bar{S} + m\bar{g} - \underbrace{2m\bar{\omega}\times\bar{v}_r}_{\text{Coriolis}}$$

r: rotating – hringsnúandi kerfi

$$\bar{g} = \bar{g}_0 - \underbrace{\bar{\omega}\times\{\bar{\omega}\times(\bar{r}+\bar{E})\}}$$

miðflótakreftir vegna hringsnúnings

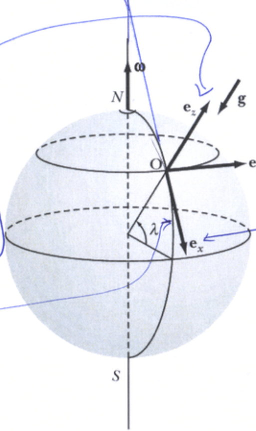
Kraftur sum virðist verta í kerfi sum er ikki tregðukerfi

Setjum $\bar{S} = 0$, og til ðe stødda hreyfingu $\bar{F}_{eff} = m\bar{a}_r$

$$\rightarrow \boxed{\bar{a}_r = \bar{g} - 2\bar{\omega}\times\bar{v}}$$

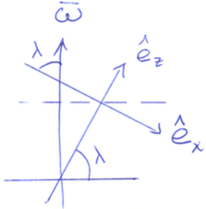
Veljum hnitakerfi þ.a.

$$\hat{e}_z \parallel (-\bar{g})$$



Gerum ráð fyrir að g sé fasti í fallinu. (g inniheldur miðflóttakraft)

N-hvel → $\begin{cases} \omega_x = -\omega\cos\lambda \\ \omega_y = 0 \\ \omega_z = \omega\sin\lambda \end{cases}$



$$\begin{cases} \bar{\omega}\cdot\hat{e}_x = -\omega\cos\lambda \\ \bar{\omega}\cdot\hat{e}_z = \omega\cos\left(\frac{\pi}{2}-\lambda\right) \\ \qquad\quad = \omega\sin\lambda \\ \bar{\omega}\cdot\hat{e}_y = 0 \end{cases}$$

Mestur hraðinn er í $\hat{e}_z$-stefnu

Coriolis → hraðar í $\hat{e}_x$ og $\hat{e}_y$ stefnur mögulegir, en

$$\boxed{\begin{aligned} \dot{x} &\simeq 0 \\ \dot{y} &\simeq 0 \\ \dot{z} &\simeq -gt \end{aligned}}$$

$$\bar{\omega} \times \bar{v}_r \approx \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ 0 & 0 & -gt \end{vmatrix} = -\hat{e}_y\,\omega g t\cos\lambda$$

$$\bar{g} = (0, 0, -g) \qquad \bar{a}_r = \bar{g} - 2\bar{\omega}\times\bar{v}$$

$$= (0,\ 2\omega g t\cos\lambda,\ -g)$$

$\rightarrow$ Kraftur Coriolis leiðir til hröðunar í $\hat{e}_y$-stefnu,
í austur

Hreyfijafnan fyrir $\hat{e}_y$-stefnu er

$$\ddot{y} = 2\omega g t\cos\lambda$$

og í $\hat{e}_z$-stefnu

$$\ddot{z} = -g$$

upphaf $y(0) = 0$, $\dot{y}(0) = 0$

$$\rightarrow y(t) = \frac{\omega g t^3}{3}\cos\lambda$$

$z(0) = h$, $\dot{z}(0) = 0$

$$\rightarrow z(t) = h - \frac{1}{2}g t^2$$

fellur í $z = 0$
á tíma $t_h = \sqrt{\frac{2h}{g}}$

$$y(t) = \frac{\omega g t^3}{3} \cos\lambda \quad \text{og} \quad t_h = \sqrt{\frac{2h}{g}}$$

$$\rightarrow y(t_h) = \frac{\omega g}{3} \cos\lambda \cdot \left(\frac{2h}{g}\right)^{3/2} = \frac{\omega}{3} \cos\lambda \cdot \sqrt{\frac{8h^3}{g}}$$

$$[y(t_h)] = \frac{L^{3/2}}{T} \frac{T}{L^{1/2}} = L \,, \quad \omega = 7.3 \cdot 10^{-5}\ \mathrm{rad/s} \,, \quad h = 100\ \mathrm{m}$$

$$d = y(t_h) - 0 \simeq 1{,}55\ \mathrm{cm} \quad \text{fyrir} \quad \lambda = 45^\circ \quad \text{(bók)}$$

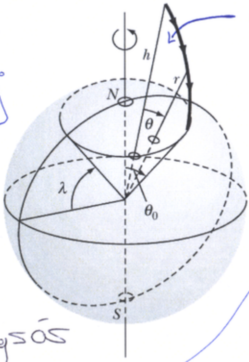
$$\simeq 0{,}96\ \mathrm{cm} \quad \text{fyrir} \quad \lambda = 64^\circ$$

Á Norðurskautinu verður ekkert frávik frá lóðlínu, og þar væri líka miðflóttakrafturinn 0 $\bar{g} = \bar{g}_0$

> Getum við sannreynt að lýsingin hér að framan í „ekki-tregðukerfi" gefi sömu niðurstöður og hreyfing í miðlægu mætti í tregðukerfi?

Massa sleppt í hæð h yfir yfirborði

{Búast má við hreyfingu eftir fleygboga með miðju jarðar í brennipunkti $\epsilon \simeq 1$}



θ er mælt frá upphafsstaðsetning massa

Þegar ögn er sleppt fær hún lárétta hraða til hægri

$$v_{hor} = r\omega\cos\lambda = (R+h)\omega\cos\lambda$$

því er hverfiþunginn um snúningsás

$$l = m r v_{hor} = m(R+h)^2\omega\cos\lambda$$

Jafna brautarinnar er

$$\frac{\alpha}{r} = 1 - \epsilon\cos\theta$$

$$\alpha = \frac{l^2}{mk}$$

Þegar $t=0$, $\theta = 0$

$$\frac{\alpha}{r} = 1 - \epsilon$$

$$\frac{\alpha}{R+h} = 1 - \epsilon$$

$$\frac{\alpha}{r} = 1 - \epsilon \cos\theta$$

$$\frac{\alpha}{R+h} = 1 - \epsilon \quad \to \quad \alpha = (1-\epsilon)(R+h)$$

$$\to \quad r = \frac{(1-\epsilon)(R+h)}{1 - \epsilon \cos\theta}$$

Áður fékkst

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{\ell}{2m} = \text{fasti}$$

$$\to \quad \frac{m}{\ell} r^2 d\theta = dt \qquad \text{og} \qquad \ell = m (R+h)^2 \omega \cos\lambda$$

$$\to \quad t - 0 = \frac{m}{\ell} \int_0^\theta r^2 d\theta = \frac{1}{\omega \cos\lambda} \int_0^\theta \left\{ \frac{1-\epsilon}{1-\epsilon \cos\theta} \right\}^2 d\theta$$

Tökum með $\underline{\theta = \theta_0}$ horn'ið þegar massinn fellur á yfirborð jarðar $(r = R)$

$$r = \frac{(1-\epsilon)(R+h)}{1-\epsilon\cos\theta} \quad \rightarrow \quad \frac{r}{R+h} = \frac{1-\epsilon}{1-\epsilon\cos\theta}$$

og $$\frac{R}{R+h} = \frac{1-\epsilon}{1-\epsilon\cos\theta_0}$$

$$\rightarrow 1 + \frac{h}{R} = \frac{1-\epsilon\cos\theta_0}{1-\epsilon} = \frac{1-\epsilon\left\{1-2\sin^2\left(\frac{\theta_0}{2}\right)\right\}}{1-\epsilon}$$

$$= 1 + \frac{2\epsilon}{1-\epsilon}\sin^2\left(\frac{\theta_0}{2}\right) \rightarrow \frac{h}{R} = \frac{2\epsilon}{1-\epsilon}\sin^2\left(\frac{\theta_0}{2}\right)$$

(8)

Brautin er nánast lóðrétt, θ breytist mjög lítið, $\theta \sim 0$

$$\rightarrow \frac{h}{R} = \frac{2\epsilon}{1-\epsilon}\sin^2\left(\frac{\theta_0}{2}\right) \simeq \frac{\epsilon\theta_0^2}{2(1-\epsilon)}$$

$$t = \frac{1}{\omega\cos\lambda}\int_0^\theta d\theta \left[\frac{1-\epsilon}{1-\epsilon\cos\theta}\right]^2 = \frac{1}{\omega\cos\lambda}\int_0^\theta \frac{d\theta}{\left[1+\frac{2\epsilon}{1-\epsilon}\sin^2\left(\frac{\theta}{2}\right)\right]^2}$$

$$\left(\frac{1}{\frac{1-\epsilon\cos\theta}{1-\epsilon}}\right)^2 = \left(\frac{1}{\frac{1-\epsilon\left(1-2\sin^2\left(\frac{\theta}{2}\right)\right)}{1-\epsilon}}\right)^2 = \left(\frac{1}{1+\frac{2\epsilon}{1-\epsilon}\sin^2\left(\frac{\theta}{2}\right)}\right)^2$$

θ lítið

$$\rightarrow t \simeq \frac{1}{\omega\cos\lambda}\int_0^\theta \frac{d\theta}{\left[1+\frac{\epsilon\theta^2}{2(1-\epsilon)}\right]^2}$$

$$\frac{\epsilon}{2(1-\epsilon)} = \frac{h}{R\theta_0^2}$$

$t(\theta=\theta_0) = T$ falltíminn

$$\to T = \frac{1}{\omega\cos\lambda}\int_0^{\theta_0} \frac{d\theta}{\left\{1+\left(\frac{h\theta^2}{R\theta_0^2}\right)\right\}^2} \simeq \frac{1}{\omega\cos\lambda}\int_0^{\theta_0} d\theta\left\{1-\frac{2h\theta^2}{R\theta_0^2}\right\}$$

$$= \frac{1}{\omega\cos\lambda}\left\{1-\frac{2}{3}\frac{h}{R}\right\}\theta_0$$

$$\to \theta_0 = \frac{\omega T\cos\lambda}{1-\frac{2h}{3R}} \simeq \omega T\cos\lambda\cdot\left\{1+\frac{2h}{3R}\right\}$$

Í fallinu snýst jörðin um ωT $\to$ punkturinn undir massanum klukkan $t=0$ færist í austur um $(TR\omega\cos\lambda)$.
Á sama tíma liggur braut massans í austur um $R\theta_0$
Því er hliðrunin í austur

$$d = R\theta_0 - R\omega T\cos\lambda$$

$$\simeq \frac{2}{3}h\omega T\cos\lambda$$

Áður sáum við að $T \simeq \sqrt{\frac{2h}{g}}$ ← nálgun hér...

$$\rightarrow d \simeq \frac{2}{3} h \omega T \cos\lambda \simeq \frac{1}{3} \omega \cos\lambda \cdot \sqrt{\frac{8h^3}{g}}$$

eins og áður

Pendúll Foucault

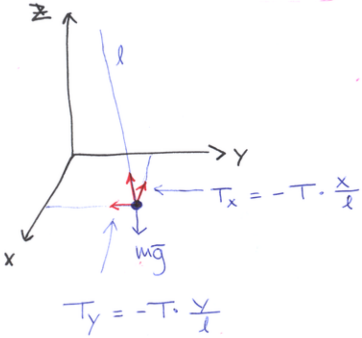
Sveiflusl étta penduls sem er rétt hengdur upp snýst vegna snúnings jarðar

frekar flókið fyrirbæri → skoðum smáar sveiflur

setjum $\hat{e}_z$ samsíða staðbundinni lóðlínu

$$\rightarrow \dot{z} << \dot{x}, \dot{y} \quad \rightarrow \quad \dot{z} \sim 0 \qquad x, y << l$$

$$\bar{a}_r = \bar{g} + \frac{\bar{T}}{m} - 2\bar{\omega} \times \bar{v}_r$$



$$T_z \simeq T$$

$$T_x = -T \frac{x}{\ell} \quad \sim \sin\theta_x$$

$$T_y = -T \frac{y}{\ell} \quad \sim \sin\theta_y$$

$$g_x = 0$$

$$g_y = 0$$

$$g_z = -g$$

$$\omega_x = -\omega \cos\lambda$$

$$\omega_y = 0$$

$$\omega_z = \omega \sin\lambda$$

$$(\bar{v}_r)_x = \dot{x}$$

$$(\bar{v}_r)_y = \dot{y}$$

$$(\bar{v}_r)_z = \dot{z} \simeq 0$$

$$\bar{\omega} \times \bar{v}_r \simeq \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ \dot{x} & \dot{y} & 0 \end{vmatrix}$$

$$= \left(-\dot{y}\omega\sin\lambda,\ \dot{x}\omega\sin\lambda,\ -\dot{y}\omega\cos\lambda\right)$$

$$\rightarrow \boxed{\begin{aligned} (a_r)_x &= \ddot{x} \simeq -\frac{T}{m}\frac{x}{l} + 2\dot{y}\omega\sin\lambda \\ (a_r)_y &= \ddot{y} \simeq -\frac{T}{m}\frac{y}{l} - 2\dot{x}\omega\sin\lambda \end{aligned}}$$

Smáar sveiflur $T \simeq mg$, setjum $\alpha^2 = \frac{T}{ml} \simeq \frac{g}{l}$
og $\omega_z = \omega\sin\lambda$

þá fæst

$$\boxed{\begin{aligned} \ddot{x} + \alpha^2 x &= 2\omega_z \dot{y} \\ \ddot{y} + \alpha^2 y &= -2\omega_z \dot{x} \end{aligned}}$$

← Tengdar 2. stigs jöfnur

þessar jöfnur má „aftengja" með (leggja saman . . . .)

$$(\ddot{x} + i\ddot{y}) + \alpha^2(x+iy) = -2\omega_z(i\dot{x} - \dot{y}) = -2i\omega_z(\dot{x} + i\dot{y})$$

og skilgreina $q \equiv x + iy$

því hneppið verður þá

$$\boxed{\ddot{q} + 2i\omega_z\dot{q} + \alpha^2 q = 0}$$

Svipuð deyfða sveiflunum

Lausnin er

$$q(t) = e^{-i\omega_z t}\left\{A e^{\sqrt{-\omega_z^2-\alpha^2}\,t} + B e^{-\sqrt{-\omega_z^2-\alpha^2}\,t}\right\}$$

Ef jörðin væri kyrr fengist

$$\ddot{q}' + \alpha^2 q' = 0 \qquad \text{því } \omega_z = 0$$

Sveiflan er miklu hraðari en snúningurinn $\alpha \gg \omega_z$

$$\rightarrow q(t) \simeq e^{-i\omega_z t}\left\{A e^{i\alpha t} + B e^{-i\alpha t}\right\}$$

Lausn óhliðraðu jöfnunnar er

$$q'(t) = x'(t) + iy'(t) = A e^{i\alpha t} + B e^{-i\alpha t}$$

$$\rightarrow \quad q(t) = q'(t)\cdot e^{-i\omega_z t}$$

$$\rightarrow x(t) + iy(t) = \left\{x'(t) + iy'(t)\right\}\cdot e^{-i\omega_z t}$$

$$= (x' + iy')\left\{\cos(\omega_z t) - i\sin(\omega_z t)\right\}$$

$$= \left\{x'\cos(\omega_z t) + y'\sin(\omega_z t)\right\} + i\left\{-x'\sin(\omega_z t) + y'\cos(\omega_z t)\right\}$$

Sama sem hneppi

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \underbrace{\begin{pmatrix} \cos(\omega_z t) & \sin(\omega_z t) \\ -\sin(\omega_z t) & \cos(\omega_z t) \end{pmatrix}}_{\text{snúningur í sléttu}} \begin{pmatrix} X'(t) \\ Y'(t) \end{pmatrix}$$

slétta pendúlsins snýst með $\boxed{\omega_z = \omega \sin\lambda}$

snýst mest á skautunum og alls ekki á miðbaug