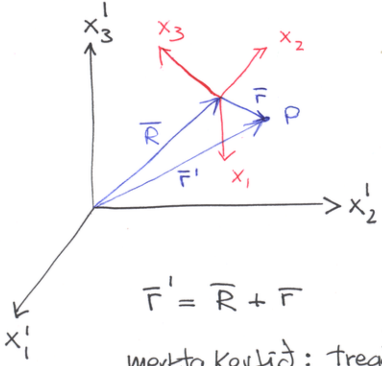
Hreyfingu lýst utan tregðukerfis

Aðallega hér kerfi sem snýst ~~miðað~~ við tregðukerfi — yfirborð jarðar

Setjum upp tvö hnitakerfi



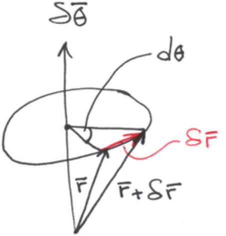
$$\bar{r}' = \bar{R} + \bar{r}$$

merkta kerfið: tregðukerfi

Ómerkta kerfið: ekki tregðukerfi

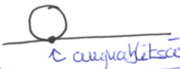
$\bar{R}$: staðsetning snúningshnit-kerfis

Örsmáar hreyfum má alltaf lýsa sem örsmáar snúning um augnabliksás



$$d\bar{r} = d\bar{\theta}\times\bar{r}$$

t.d. hjól sem veltur



Hér, ef x_i-kerfið snýst um $\delta\hat{\theta}$

$$(d\bar{r})_{fixed} = d\bar{\theta}\times\bar{r}$$

Gerum ráð fyrir að P sé fast í x_i-kerfi

$$\rightarrow \left(\frac{d\bar{r}}{dt}\right)_{fixed} = \frac{d\bar{\theta}}{dt} \times \bar{r} \quad = \bar{\omega} \times \bar{r}$$

Ef P er með hraða $\left(\frac{d\bar{r}}{dt}\right)_{rot}$ miðað við x_i-kerfi

$$\rightarrow \left(\frac{d\bar{r}}{dt}\right)_{fixed} = \left(\frac{d\bar{r}}{dt}\right)_{rot} + \bar{\omega} \times \bar{r}$$

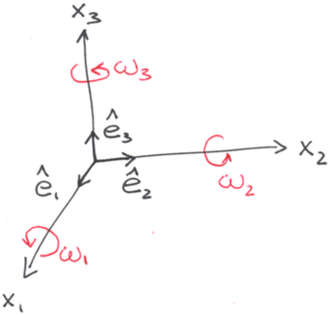
<u>skoðum aðeins betur</u>

Setjum $\quad \bar{r} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3$

og $\bar{R} = 0 \leftarrow$ hnitakerfin eru með sama upphafspunkt

$$\rightarrow \left(\frac{d\bar{r}}{dt}\right)_{fixed} = \frac{d}{dt}\left\{\sum_i x_i\hat{e}_i\right\} = \sum_i \left\{\dot{x}_i\hat{e}_i + x_i\dot{\hat{e}}_i\right\}$$

$$\left(\frac{d\bar{r}}{dt}\right)_{fixed} = \dot{\bar{r}}_{rot} + \sum_i x_i \dot{\hat{e}}_i$$



$$\frac{d\hat{e}_1}{dt} = \omega_3 \hat{e}_2 - \omega_2 \hat{e}_3$$

$$\frac{d\hat{e}_2}{dt} = -\omega_3 \hat{e}_1 + \omega_1 \hat{e}_3$$

$$\frac{d\hat{e}_3}{dt} = \omega_2 \hat{e}_1 - \omega_1 \hat{e}_2$$

$$\boxed{\dot{\hat{e}}_i = \bar{\omega} \times \hat{e}_i}$$

$$\rightarrow \left(\frac{d\bar{r}}{dt}\right)_{fixed} = \dot{\bar{r}}_{rot} + \sum_i \bar{\omega} \times x_i \hat{e}_i = \dot{\bar{r}}_{rot} + \bar{\omega} \times \bar{r}$$

Almennt gildir fyrir vigur $\overline{Q}$

$$\boxed{\left(\frac{d\overline{Q}}{dt}\right)_{fixed} = \left(\frac{d\overline{Q}}{dt}\right)_{rot} + \overline{\omega}\times\overline{Q}}$$

T.d. fyrir hornhröðun

$$\left(\frac{d\overline{\omega}}{dt}\right)_{fixed} = \left(\frac{d\overline{\omega}}{dt}\right)_{rot} + \overbrace{\overline{\omega}\times\overline{\omega}}^{=0} = \dot{\overline{\omega}}$$

hornhröðunin er sú sama í báðum kerfum

Byrjum aftur með

$$\overline{r}' = \overline{R} + \overline{r}$$

$$\left(\frac{d\overline{r}'}{dt}\right)_{fixed} = \left(\frac{d\overline{R}}{dt}\right)_{fixed} + \left(\frac{d\overline{r}}{dt}\right)_{fixed}$$

en

$$\left(\frac{d\overline{r}}{dt}\right)_{fixed} = \left(\frac{d\overline{r}}{dt}\right)_{rot} + \overline{\omega}\times\overline{r}$$

$$\left(\frac{d\overline{r}'}{dt}\right)_{fixed} = \left(\frac{d\overline{R}}{dt}\right)_{fixed} + \left(\frac{d\overline{r}}{dt}\right)_{rot} + \overline{\omega}\times\overline{r}$$

$$\overline{v}_f \equiv \dot{\overline{r}}_f \equiv \left(\frac{d\overline{r}'}{dt}\right)_{fixed}$$

$$\overline{V} \equiv \dot{\overline{R}}_f \equiv \left(\frac{d\overline{R}}{dt}\right)_{fixed}$$

$$\overline{v}_r \equiv \dot{\overline{r}}_{rot} \equiv \left(\frac{d\overline{r}}{dt}\right)_{rot}$$

$$\rightarrow \boxed{\overline{v}_f = \overline{V} + \overline{v}_r + \overline{\omega}\times\overline{r}}$$

$$\left(\frac{d\bar{r}'}{dt}\right)_{fixed} = \left(\frac{dR}{dt}\right)_{fixed} + \left(\frac{d\bar{r}}{dt}\right)_{rot} + \bar{\omega} \times \bar{r}$$

$$\boxed{\bar{v}_f = \bar{V} + \bar{v}_r + \bar{\omega} \times \bar{r}}$$

$\bar{v}_f$: Hraði m.v. fasta kerfið

$\bar{V}$: Hraði miðju snúningskerfis m.v. fasta kerfið

$\bar{v}_r$: Hraði m.v. snúningskerfi

$\bar{\omega}$: Snúningur snúningskerfis

$\bar{\omega} \times \bar{r}$: Hraði vegna snúnings snúningskerfis

Gervikraftar

Aðeins í tregðukerfi gildir

$$\bar{F} = m\bar{a}$$

því er hægt að finna

$$\bar{F} = m\bar{a}_f = m\left(\frac{d\bar{v}_f}{dt}\right)_{fixed}$$

$$\left(\frac{d\bar{v}_r}{dt}\right)_{fixed} = \left(\frac{d\bar{v}_r}{dt}\right)_{rot} + \bar{\omega}\times\bar{v}_r$$

$$= \bar{a}_r + \bar{\omega}\times\bar{v}_r$$

hröðun í snúningskerfinu

$$\left(\frac{d\bar{v}_f}{dt}\right)_{fixed} = \underbrace{\left(\frac{d\bar{V}}{dt}\right)_{fixed}}_{=\ddot{\bar{R}}_f} + \left(\frac{d\bar{v}_r}{dt}\right)_{fixed} + \dot{\bar{\omega}}\times\bar{r} + \bar{\omega}\times\left(\frac{d\bar{r}}{dt}\right)_{fixed}$$

$$\bar{\omega}\times\left(\frac{d\bar{r}}{dt}\right)_{fixed} = \bar{\omega}\times\left(\frac{d\bar{r}}{dt}\right)_{rot} + \bar{\omega}\times(\bar{\omega}\times\bar{r})$$

$$= \bar{\omega}\times\bar{v}_r + \bar{\omega}\times(\bar{\omega}\times\bar{r})$$

samantekið

$$\overline{F} = m\overline{a}_f = m\ddot{\overline{R}}_f + m\overline{a}_r + m\overline{\omega}\times\overline{v}_r + m\dot{\overline{\omega}}\times\overline{r} + m\overline{\omega}\times\overline{v}_r + m\overline{\omega}\times(\overline{\omega}\times\overline{r})$$

$$= m\ddot{\overline{R}}_f + m\overline{a}_r + m\dot{\overline{\omega}}\times\overline{r} + m\overline{\omega}\times(\overline{\omega}\times\overline{r}) + 2m\overline{\omega}\times\overline{v}_r$$

fyrir athuganda í snúningskerfinu

$$\boxed{\overline{F}_{eff} \equiv m\overline{a}_r = \overline{F} - m\ddot{\overline{R}}_f - m\dot{\overline{\omega}}\times\overline{r} - m\overline{\omega}\times(\overline{\omega}\times\overline{r}) - 2m\overline{\omega}\times\overline{v}_r}$$

Hliðrunarhröðun

miðflóttakraftur

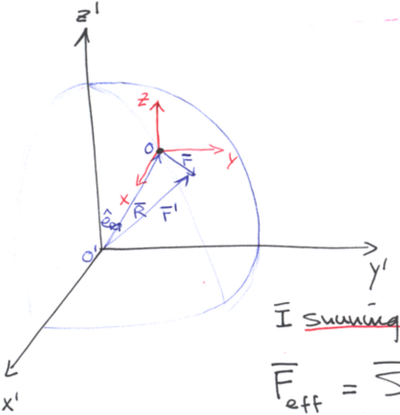
Hornhröðun

Coriolis kraftur (1835)

snúningskerfis m.v. fastakerfið

Gervikraftar

Hreyfing m.v. jörð



Mælt í **fasta kerfinu**

$$\bar{F} = \bar{S} + m\bar{g}_0$$

$\bar{S}$: ytri krafter, rafsegul, viðnám, ...

$$\bar{g}_0 = -G\frac{M_E}{R^2}\hat{e}_R$$

Í **snúningskerfinu**

$$\bar{F}_{eff} = \bar{S} + m\bar{g}_0 - m\ddot{\bar{R}}_f \underbrace{- m\dot{\bar{\omega}} \times \bar{r}}_{\sim 0} - m\bar{\omega}\times(\bar{\omega}\times\bar{r}) - 2m\bar{\omega}\times\bar{v}$$

en

$$\left(\frac{d\bar{Q}}{dt}\right)_f = \left(\frac{d\bar{Q}}{dt}\right)_r + \bar{\omega}\times\bar{Q}$$

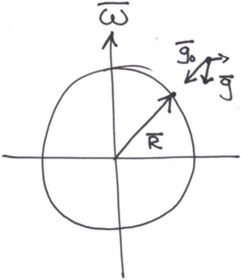
$$\hookrightarrow \ddot{\bar{R}}_f = 0 + \bar{\omega}\times\dot{\bar{R}}_f = 0 + \bar{\omega}\times(\bar{\omega}\times\bar{R})$$

$$\rightarrow \bar F_{eff} = \bar S + m\bar g_0 - m\bar\omega\times\left(\bar\omega\times(\bar r+\bar R)\right) - 2m\bar\omega\times\bar v_r$$

Því er heppilegt að skilgreina

$$\bar g = \bar g_0 - \underbrace{\bar\omega\times\left\{\bar\omega\times(\bar r+\bar R)\right\}}$$

{ miðflóttakrafturinn hefur áhrif á þyngdarhröðunina
breytir lögun jarðar



$-\bar\omega\times(\bar\omega\times\bar R)$

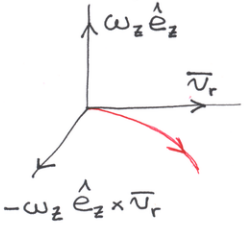
- Sterkastur við miðbaug
- Hverfur á skautum
- Hafið er lárétt m.v. $\bar g$, ekki $\bar g_0$
- Mælanlegt með pendúli

$\omega^2 R \sim 0{,}034\ m/s^2$

Kraftur Coriolis á jörð

$-2m\bar{\omega} \times \bar{v}_r$

Í snertisléttu á jörð, norðurhvel



Sterkastur við norðurskaut minnkar í átt að miðbaugi þar sem hann hverfur

↑

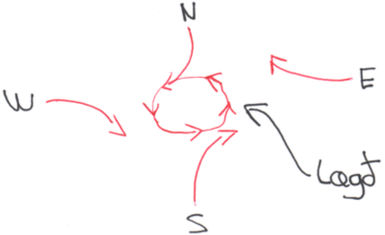
Snýst við á suðurhveli

Í snertisléttu á Norðurhveli → hægri beygja

Í snertisléttu á Suðurhveli → vinstri beygja

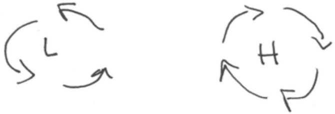
N-hvel

Loft flæði frá háþrýstingi að lægum



N-hvel

séð ofan frá á snertisléttu



Öfugt á suðurhveli

Engar sterkar lægðir á miðbaug

Staðvindar

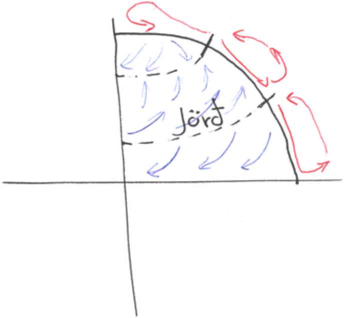
Eru flóknari vegna hitunar frá yfirborði

Rétt norðan Íslands

NV-vindar

Rétt sunnan Ísland

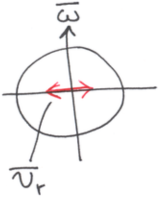
suðvestan-vindur

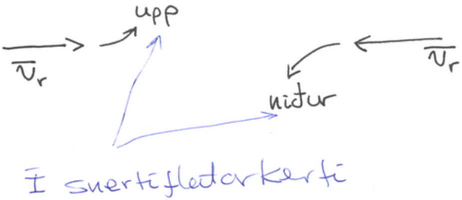


Speglað fyrir S-hvel

Ein hreyfing við yfirborð er ekki bara í snertisléttu

T.d. á miðbaug er $\omega_z = 0$, en $\underbrace{\bar{\omega} \times \bar{v}_r \neq 0}$





Eldflaugar fyrir sporbraut eru sendar austur

Eötvös-hrif