Ringl

Lítum á deyfðan og þvingaðan sveifil

Hann er þvingaður með vægi $N_d \cos(\omega_d t)$

$$N = I \frac{d^2\theta}{dt^2} = I\ddot{\theta} = -b\dot{\theta} - mgl \sin\theta + N_d \cos(\omega_d t)$$

Hreyfijafnan er

$I = ml^2$

$$\boxed{\ddot{\theta} + \frac{b}{ml^2}\dot{\theta} + \frac{g}{l}\sin\theta = \frac{N_d}{ml^2}\cos(\omega_d t)}$$

Stöllum $s = t\omega_0$, $\omega_0 = \sqrt{\frac{g}{l}}$

$$x' = \frac{dx}{ds} = \frac{d\theta}{dt}\frac{dt}{ds} = \frac{d\theta}{dt}\frac{1}{\omega_0}$$

Köllum

$x = \theta$

$$F = \frac{N_d}{ml^2\omega_0^2}$$

$$C = \frac{b}{ml^2\omega_0}$$

$$x'' = \frac{\ddot{\theta}}{\omega_0^2}$$

$$\omega_d t = \frac{\omega_d}{\omega_0} s = \omega s$$

Hreyfijafnan er þá

$$x'' + Cx' + \sin(x) = F\cos(\omega s)$$

Tölulega lausn, breytum í tvær 1. stigs jöfnur

$$y_1 = x$$
$$y_2 = x'$$

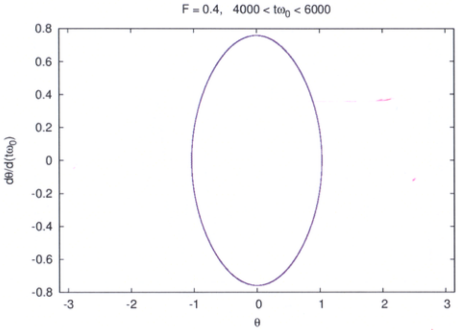
$$\boxed{\begin{aligned} y_1' &= x' = y_2 \\ y_2' &= x'' = -Cx' - \sin(x) + F\cos(\omega s) = -Cy_2 - \sin(y_1) + F\cos(\omega s) \end{aligned}}$$

Setjum $C = 0{,}05$ og <u>breytum ~~aðeins~~</u> F

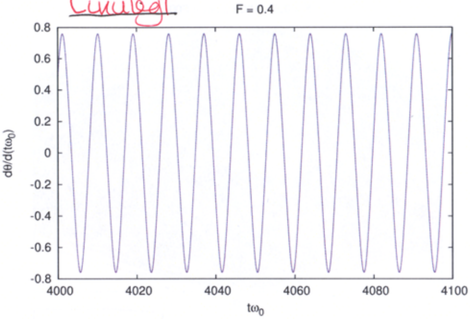
$\omega = 0{,}7$

og keyrum upplýsingum um sveiflu lausnina

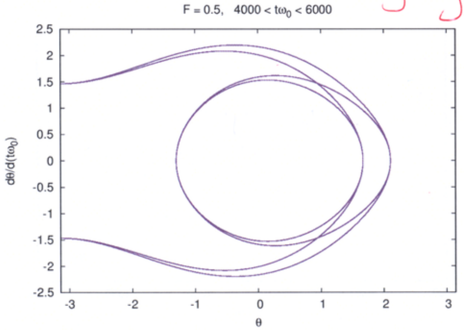
③



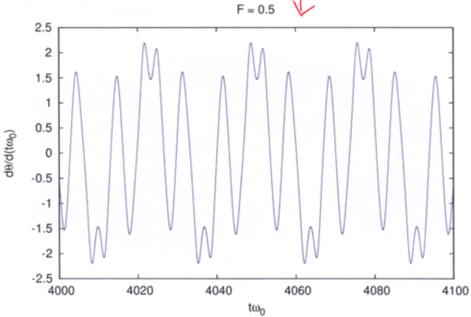
Línulegt

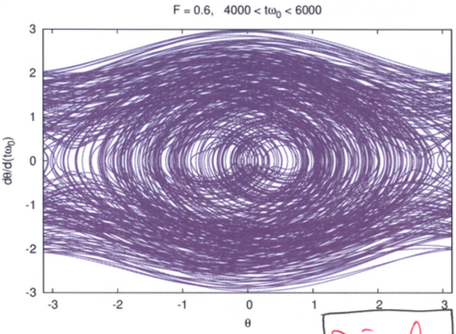


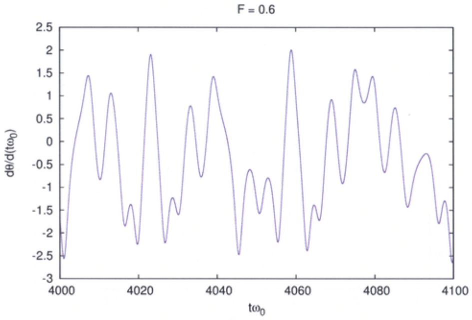
Regulegt



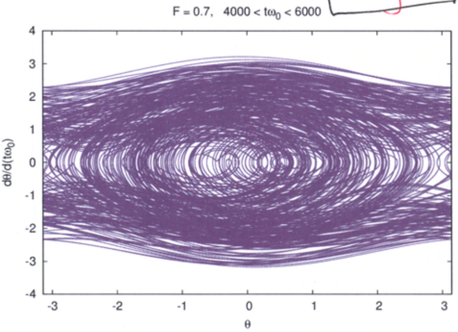
Lotutvöföldun

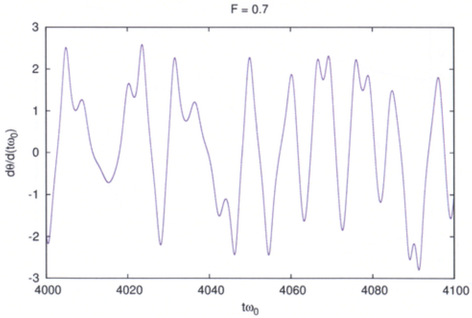



F = 0.6,   4000 < tω$_0$ < 6000
dθ/d(tω$_0$)
θ


F = 0.6
dθ/d(tω$_0$)
tω$_0$

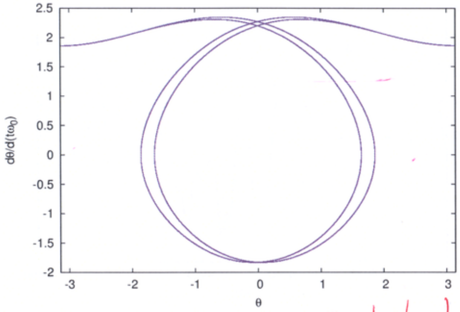
Ringl


F = 0.7,   4000 < tω$_0$ < 6000
dθ/d(tω$_0$)
θ

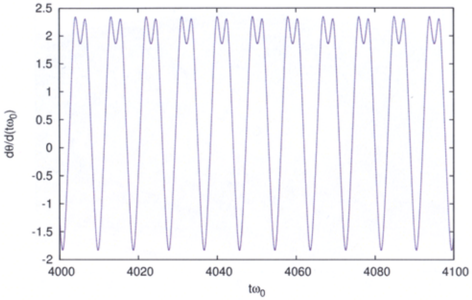

F = 0.7
dθ/d(tω$_0$)
tω$_0$

5

F = 0.8, $4000 < t\omega_0 < 6000$

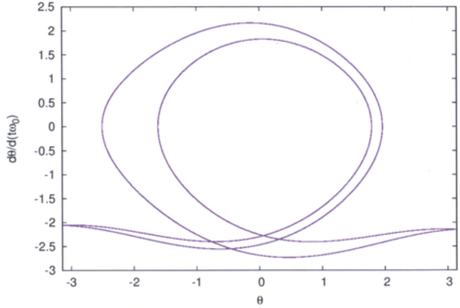


F = 0.8

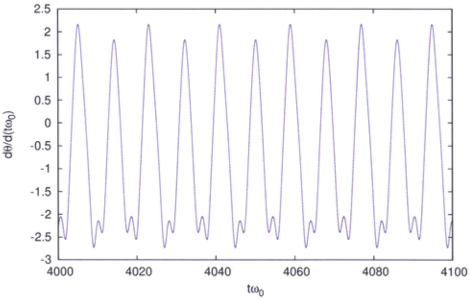


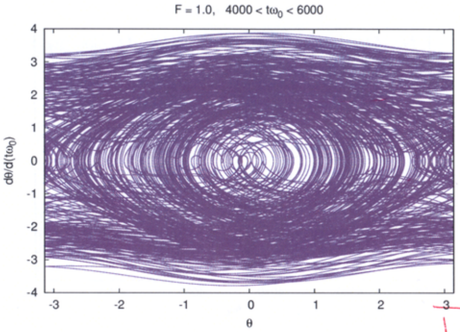
Regulært

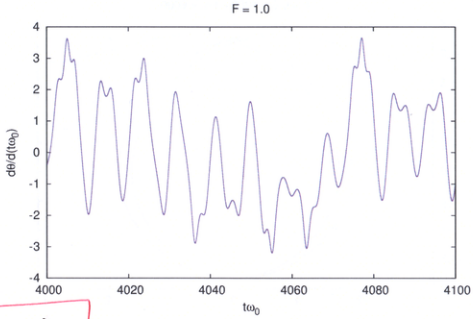
F = 0.9, $4000 < t\omega_0 < 6000$



F = 0.9







Ringlaet

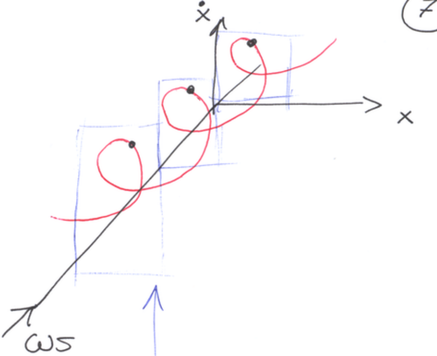
Hér takast á tveir tímaskalar sem ákvarðast af ω_d og ω_0

Fyrir lítið útslag ($F \le 0{,}4$) ræður þvingunar tíðnin

Því skoðaði Poincaré snið í tíma af $(\dot{x}, x)$ eða $(\dot{\theta}, \theta)$ með

$$\omega s = 2\pi n$$

þá ætti lotubundin hreyfing með einfalda lotu að lenda alltaf á sama stað í $(\dot{\theta}, \theta)$



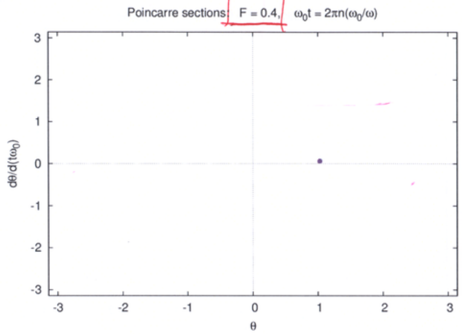
Poincaré-snið

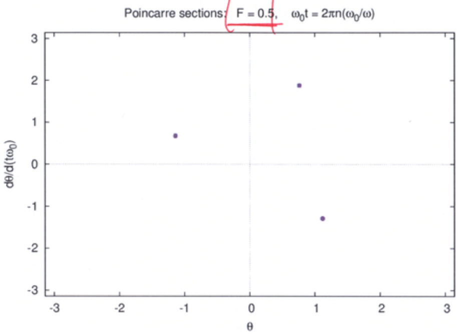
Skoðum fyrir sveifilinn

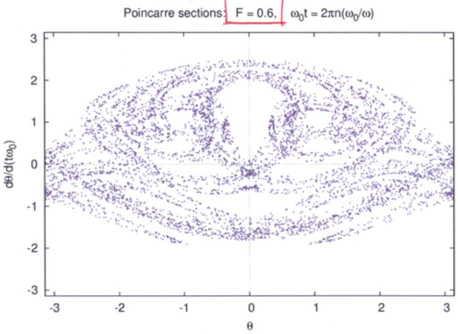
$\omega s = 2\pi n$, $s = t\omega_0$

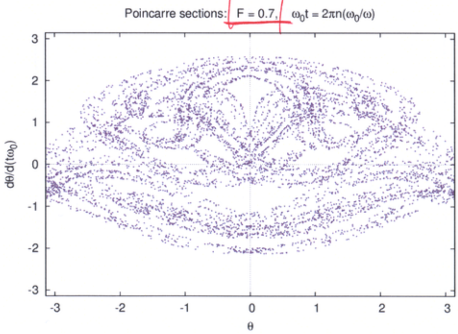
$\omega(\omega_0 t) = 2\pi n$

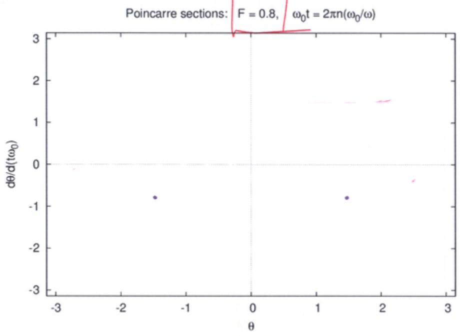
$$\boxed{\omega_0 t = 2\pi n/\omega = 2\pi n \frac{\omega_0}{\omega_d}}$$

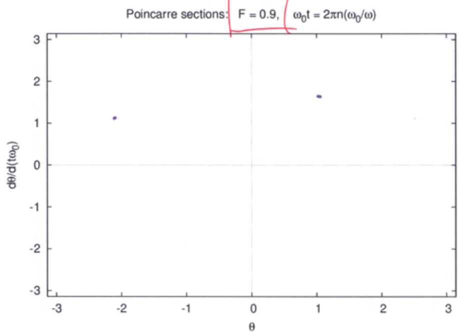

Poincarre sections: F = 0.4, $\omega_0 t = 2\pi n(\omega_0/\omega)$
$d\theta/d(t\omega_0)$
θ

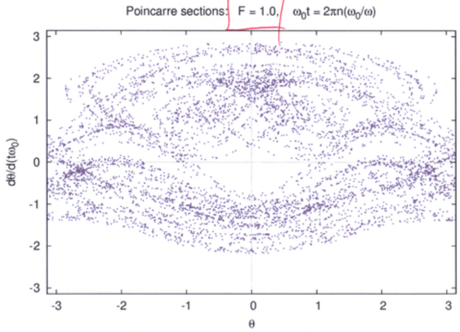

Poincarre sections: F = 0.5, $\omega_0 t = 2\pi n(\omega_0/\omega)$
$d\theta/d(t\omega_0)$
θ


Poincarre sections: F = 0.6, $\omega_0 t = 2\pi n(\omega_0/\omega)$
$d\theta/d(t\omega_0)$
θ


Poincarre sections: F = 0.7, $\omega_0 t = 2\pi n(\omega_0/\omega)$
$d\theta/d(t\omega_0)$
θ


Poincare sections: F = 0.8, $\omega_0 t = 2\pi n(\omega_0/\omega)$
dθ/d(tω0)
θ


Poincare sections: F = 0.9, $\omega_0 t = 2\pi n(\omega_0/\omega)$
dθ/d(tω0)
θ


Poincare sections: F = 1.0, $\omega_0 t = 2\pi n(\omega_0/\omega)$
dθ/d(tω0)
θ

Hvað með skammtakerfi

Sígild rafeind í segulsviði fer á hringhreyfingu í sléttu þvert á segulsviðið

Geislinn er háður B, $r \sim \frac{1}{B}$

Rafeind lýst með skammtafræði í þverstæðu segulsviði fær orkuróf

$$E_n = \hbar\omega_c\left(n+\frac{1}{2}\right)$$

$$\hbar\omega_c = \frac{\hbar e B}{m}$$

$$n = 0, 1, 2, \ldots\ldots$$

Landau-stig

stjált orkuróf

Í kjörkerfi eru Landau-stigin óendanlega grönn

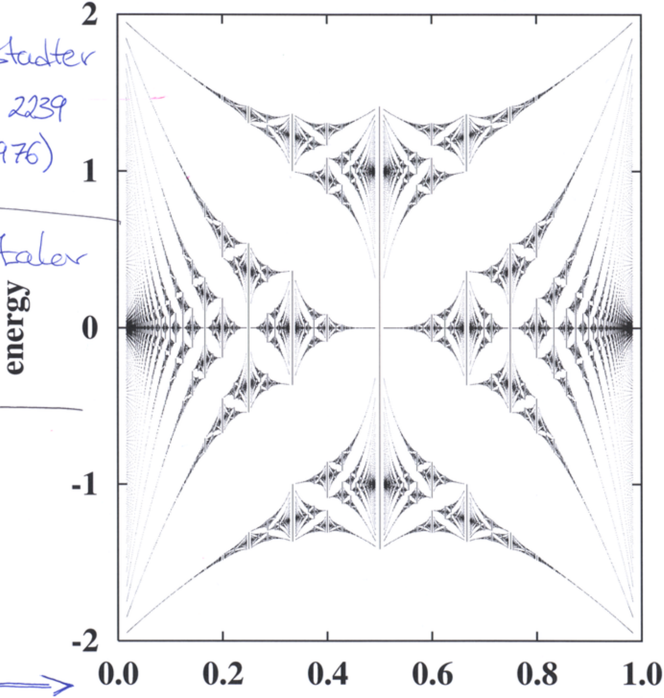
Ef rafeindin er í tvílotu bandmætti klofnar hvert Landau-stig sem brotafall af $1/B$

↳ $\boxed{\text{Orkuróf Hofstadter}\quad \text{Fiðrildi Hofstadter}}$

sjá myndir

Fundið of
Douglas R. Hofstadter
Phys. Rev. $\underline{14}$, 2239
(1976)

Tveir lengdarskalar
<u>takast</u> á
l_B og a



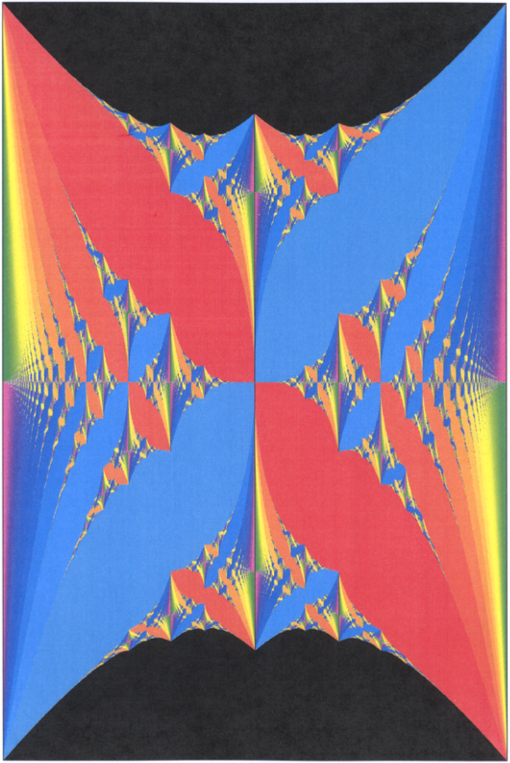
Φ/Φ_0

$\sim$ 1/segulflæði $\Longrightarrow$

einn flæði-
skammtur
um lotu-
einingu

fyrir $\frac{\Phi}{\Phi}$ rauða
tölu klofnar
Landau stígið
upp í endalausan
fjölda stiga

Orka



Wikipedia

$\frac{1}{\text{segulflæði}}$

Varpanir

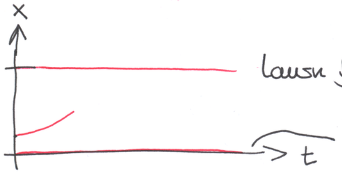
Mjög einfalt dæmi

vaxtarjafna ((logistic)(Verhulst))

fyrir hópa......

$$\frac{dx}{dt} = \alpha(1-x)x$$

heftur vöxtur



lausn fyrir $x(0)=1$

fyrir $x(0)=0$

hefur lausn

$$x(t) = \frac{1}{1+\left[\frac{1}{x(0)}-1\right]e^{-\alpha t}}$$

má umskrifa í ítrekunarjöfnuna

$$x_{n+1} = \alpha(1-x_n)x_n$$

$$\frac{dx}{dt} = \alpha(1-x)x$$

$$x_{n+1} - x_n = \alpha x_n(1-x_n)\delta t$$

$$x_{n+1} = x_n + \alpha x_n(1-x_n)\delta t$$

$$= x_n\{1+\alpha\delta t - \alpha\delta t\, x_n\}$$

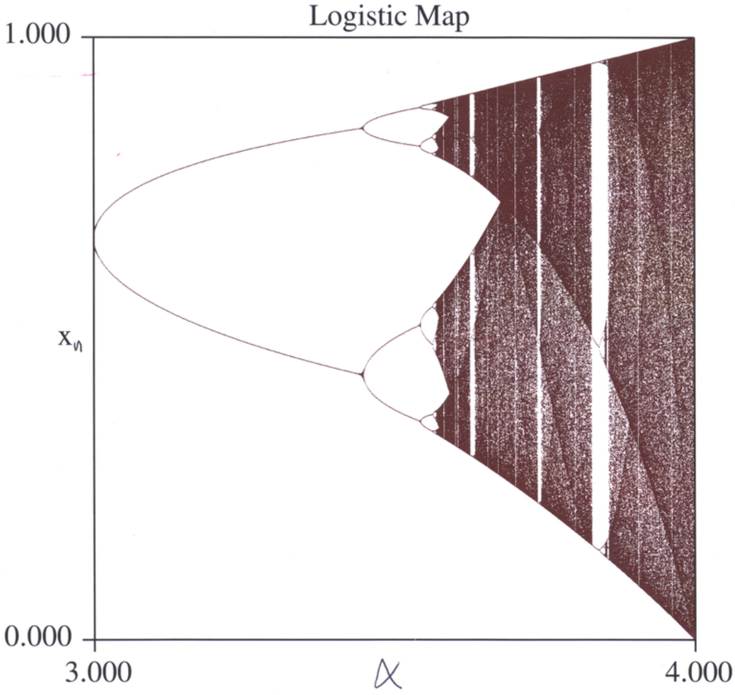
Skilgreinum

$$y_n = \frac{\alpha\delta t}{1+\alpha\delta t}x_n, \qquad \boxed{r = 1+\alpha\delta t}$$

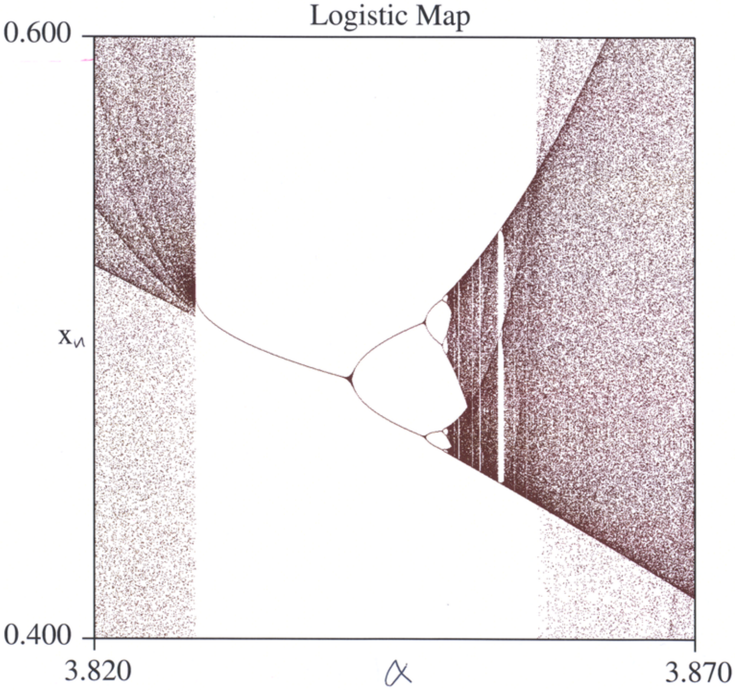
$$\rightarrow y_{n+1} = \underbrace{\frac{\alpha\delta t}{1+\alpha\delta t}x_n}_{y_n}\left[1+\alpha\delta t - (1+\alpha\delta t)y_n\right]$$

$$= r\,y_n(1-y_n)$$

Anders Sandvik Boston University



Anders Sandvik Boston University



Lyapunov vísar

upphafsástönd með litlum mun

x_0

$x_0 + \epsilon$

Vísir Lyapunovs λ er stuðull fyrir meðal veldisvísisvöxt fyrir ástöndin á einingartíma. Eftir n ítretanir er munur

$$d_n = \epsilon e^{n\lambda}$$

lítum á vörpun

$$x_{n+1} = f(x_n)$$

$\lambda < 0$: samleitni

$\lambda > 0$: sundurleitni

setjum $d_0 = \epsilon$

$$d_1 = f(x_0+\epsilon) - f(x_0) \approx \epsilon \left.\frac{df}{dx}\right|_{x_0}$$

skilgreiningin gefur þá

$$d_n = f^n(x_0+\epsilon) - f^n(x_0) = \epsilon e^{n\lambda}$$

þ.s. $f^n(x_0) = f(f(\cdots f(x_0)\cdots))$

$$\rightarrow \ln\left\{\frac{f^n(x_0+\epsilon) - f^n(x_0)}{\epsilon}\right\} = n\lambda$$

ϵ er smátt $\rightarrow$

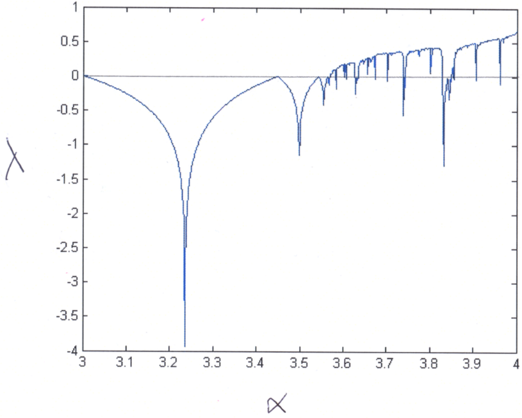
$$\lambda = \frac{1}{n}\ln\left\{\frac{f^n(x_0+\epsilon) - f^n(x_0)}{\epsilon}\right\}$$

$$= \frac{1}{n}\ln\left|\left.\frac{df^n(x)}{dx}\right|_{x_0}\right|$$

$$\left.\frac{df^n(x)}{dx}\right|_{x_0} = \left.\frac{df}{dx}\right|_{x_{n-1}} \cdot \left.\frac{df}{dx}\right|_{x_{n-2}} \cdot \ldots \cdot \left.\frac{df}{dx}\right|_{x_0}$$

$$\rightarrow \boxed{\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln\left|\frac{df(x_i)}{dx}\right|}$$

skoðum myndir fyrir „logistic map"



$\lambda > 0$ engin samlutni

Fakulteta za Strojništvo, Univerza v Ljubljani