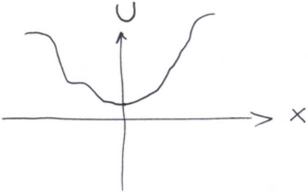
Sveiflur

Atóm í grind,

bíll á fjöðrum, ...

Allt um kring eru kerfi sem sveiflast, t.d. um lágmark í stöðuorku



Kerfið truflað frá jafnvægispunktinum í $x=0$ hefur kraft sem leitast við að koma því aftur í jafnvægi

$$F(x) = F_0 + x F'(0) + \frac{x^2}{2!} F''(0) + \dots$$

Jafnvægi í $x=0 \rightarrow F_0 = 0$

Því er lægsta nálgunin að

$$\boxed{F(x) = -kx}$$

með $$k = \left.\frac{dF}{dx}\right|_{x=0} = F'(0)$$

Við skoðum

Sveiflur

deyfingu

Örvun

fasarúm

Byrjum með línulegt F(x), en nálgumst ólínuleg kerfi (næsti kafli)

Í einni vídd er hreyfijafnan

$$\boxed{\ddot{x} + \omega_0^2 x = 0}$$

þar sem ω_0^2 er grunntíðni kerfisins. Kerfið er hreintóna sveifill, sígildur

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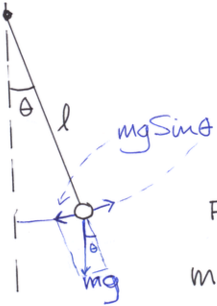
Tökum sem sýnidæmi einfaldan sveifil, stór útslög getur haldið honum línulegum, eða gert ólínul.



$$F = -mg\sin\theta$$

$$ma = -mg\sin\theta$$

$$a = l\ddot{\theta}$$

því er hreyfijafnan

$$\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0} \quad (1)$$

og fyrir lítil horn

$$\sin\theta \approx \theta$$

$$\rightarrow \boxed{\ddot{\theta} + \omega_0^2\theta = 0} \quad (2)$$

með $\omega_0 = \sqrt{\frac{g}{l}}$

línulegi sveifillinn er eins og hreintóna sveifill með lausn

$$\theta(t) = \theta_0 \sin(\omega_0 t - \delta)$$

eða

$$\theta(t) = \theta_0 \cos(\omega_0 t - \phi)$$

þ.s. útslagið θ_0 og fasahornið ákvarðast af upphafsskilyrðum

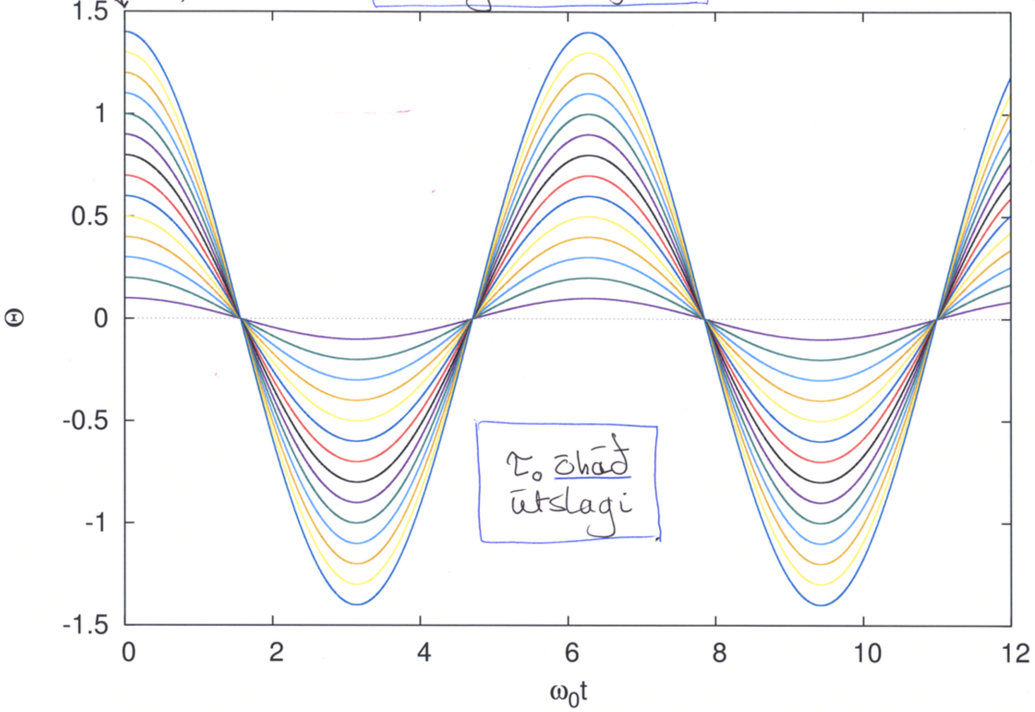
Jafna (1) hefur líka þekkta lausn í sporbaugsföllum, en hér berum við saman tölulegar lausnir þeirra fyrir vaxandi útslag

Fyrir línulega sveifilinn fæst lotulengd

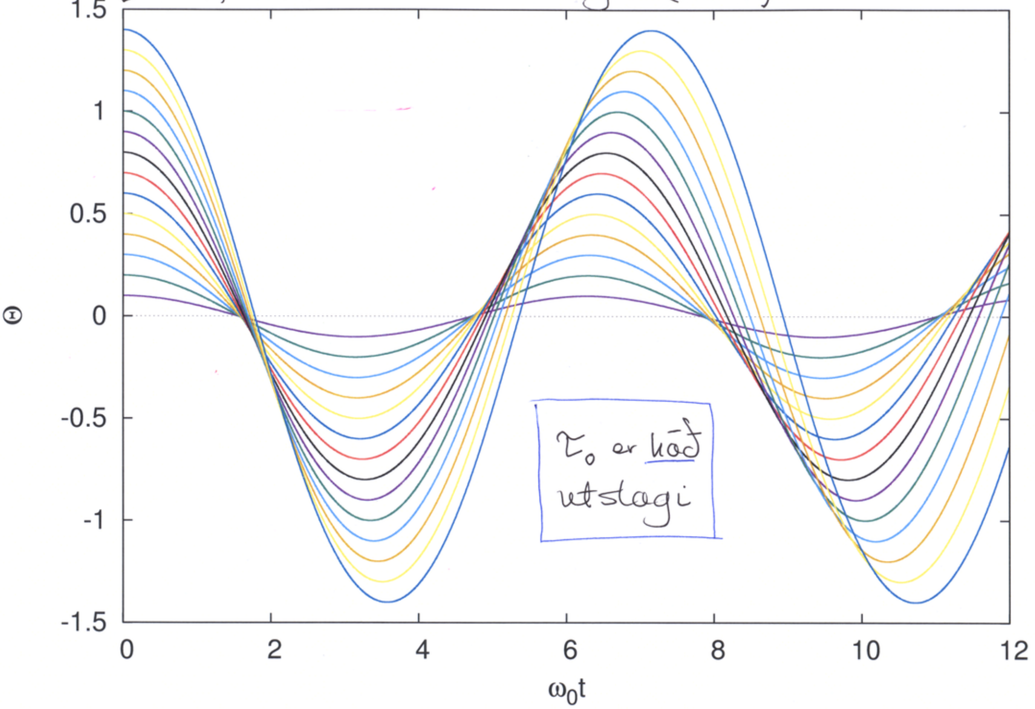
$$\omega_0\tau_0 = 2\pi$$

$$\rightarrow \tau_0 = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega_0}$$

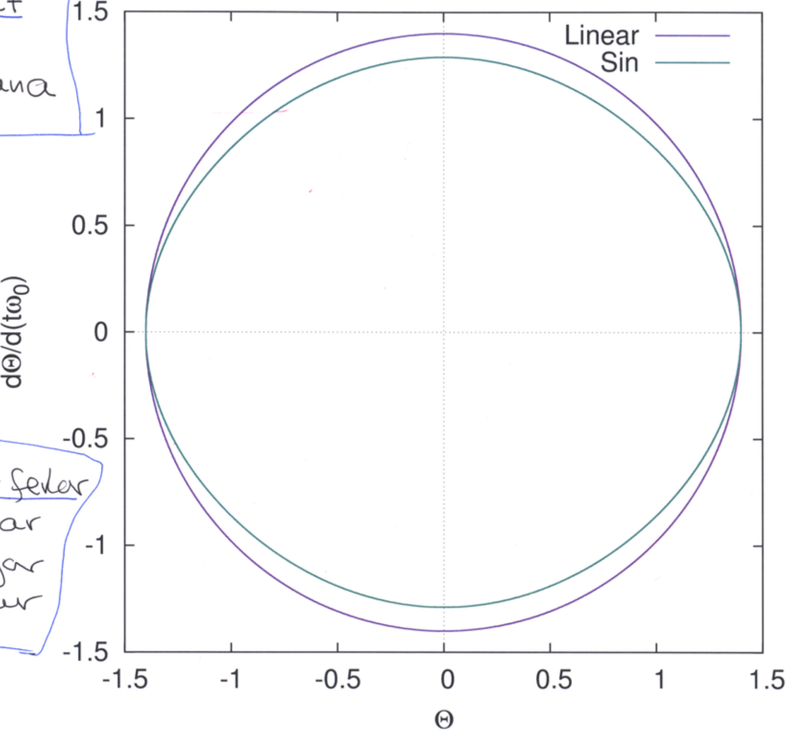
$\hookrightarrow \theta(0),\ \dot{\theta}(0)=0$ $\boxed{\text{línulegur sveifill}}$ (4)



↙ $\theta(0)$, $\dot{\theta}(0) = 0$ ólínulegur ($\sin\theta$)



Fasarit fyrir sveiflana

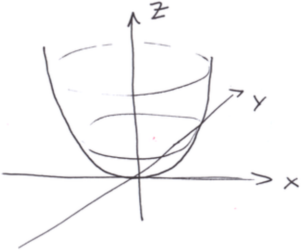


Lokaðir ferlar ódeyfðar reglulegar sveiflur

Hreintóna sveiflur í 2D

$$\ddot{x} + \omega_x^2 x = 0$$
$$\ddot{y} + \omega_y^2 y = 0$$

$$x(t) = A\cos(\omega_x t - \alpha)$$
$$y(t) = B\cos(\omega_y t - \beta)$$

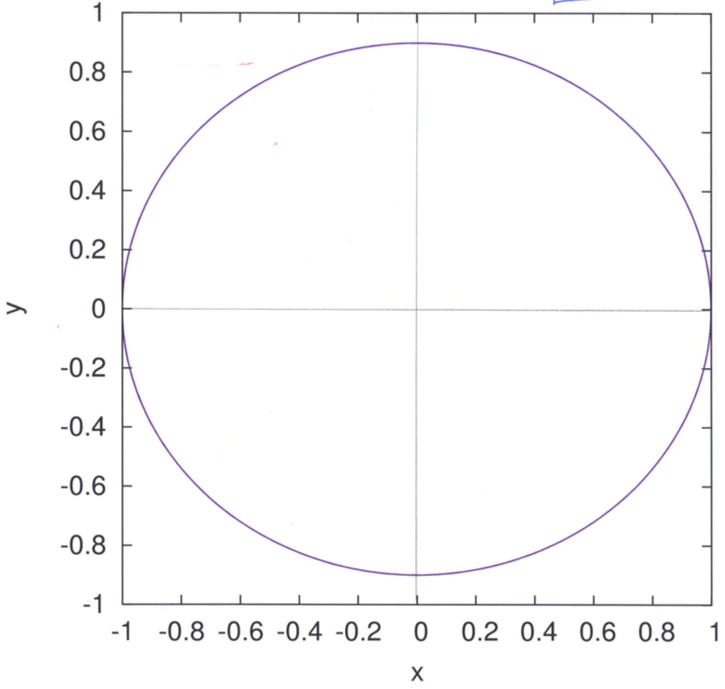


Sveiflurnar í x- og y- átt eru ótengdar

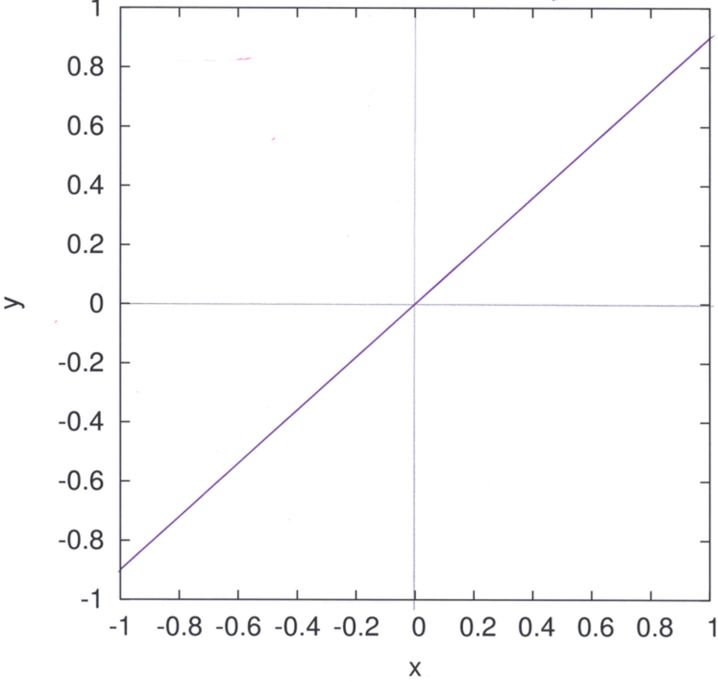
en fasamunur þeirra $\delta = \alpha - \beta$ og tíðnihlutfall leiða til

Lissajous - ferla í tvívíða x-y-sléttunni

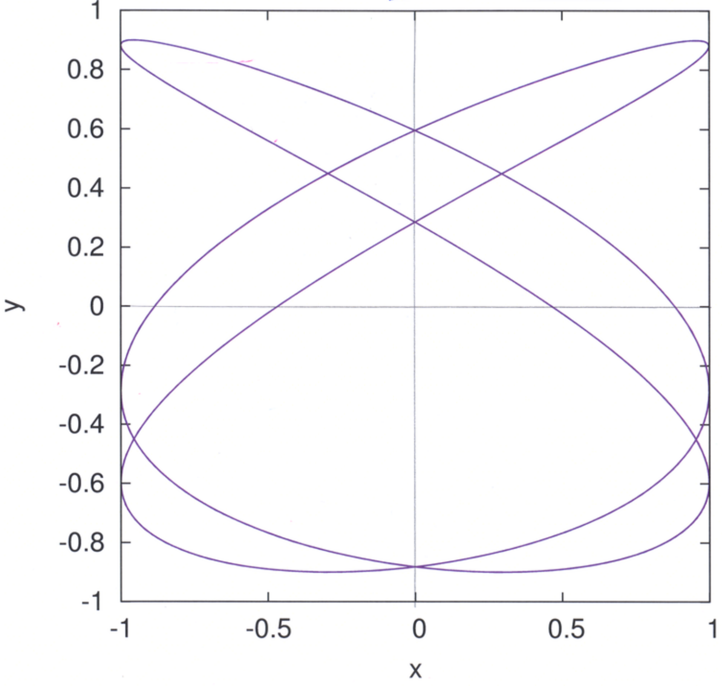
$A = 1, \; B = 0.9, \; \omega_x = 1.0\omega_y, \; \boxed{\delta = \pi/2}$



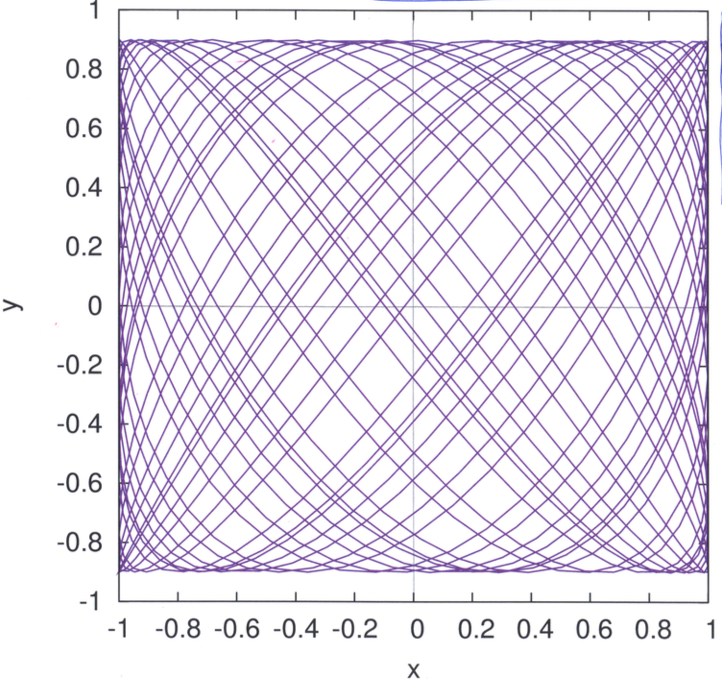
$A = 1$, $B = 0.9$, $\omega_x = 1.0\omega_y$, $\boxed{\delta = 0.0}$



$A = 1$, $B = 0.9$, $\boxed{\omega_x = 1.5\omega_y, \quad \delta = 0.2}$



$A = 1, \; B = 0.9, \; \boxed{\omega_x = \text{sqrt}(2)\omega_y, \quad \delta = 0.2}$



Og heldur áfram að fylla svæðið

Deyfðar sveiflur

Könnum sveifil með viðnámslið í réttu hlutfalli við ferð

$$\boxed{\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2\theta = 0}$$

Þegar lausn á forminu e^{rt} er reynd finnast tvær óháðar lausnir sem taka má saman sem (línuleg jafna)

$$\boxed{\theta(t) = e^{-\beta t}\left\{A_1 e^{\alpha t} + A_2 e^{-\alpha t}\right\}}$$

með

$$\alpha = \sqrt{\beta^2 - \omega_0^2}$$

Þrjú mismunandi tilvik

1. $\omega_0^2 > \beta^2 \to \alpha$ er þvertala

Köllum $\omega_1^2 = \omega_0^2 - \beta^2$ og lausnin verður

$$\theta(t) = e^{-\beta t}\left\{A_1 e^{i\omega_1 t} + A_2 e^{-i\omega_1 t}\right\}$$

sem má umrita

$$\theta(t) = A e^{-\beta t}\cos(\omega_1 t - \delta)$$

Lausnin sýnir deyfða sveiflu og kallast því vandeyfð takið eftir að tíðnin hliðrast vegna deyfingar

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2}$$

og sveiflurnar fá meðalævi

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② $\omega_0^2 < \beta^2$ α er rauntala

$$\Theta(t) = e^{-\beta t}\left\{A_1 e^{\alpha t} + A_2 e^{-\alpha t}\right\}$$

$$\alpha = \sqrt{\beta^2 - \omega_0^2}$$

Engar sveiflur, ofdeyfðar sveiflur

③ $\omega_0^2 = \beta^2$ $\alpha = 0$

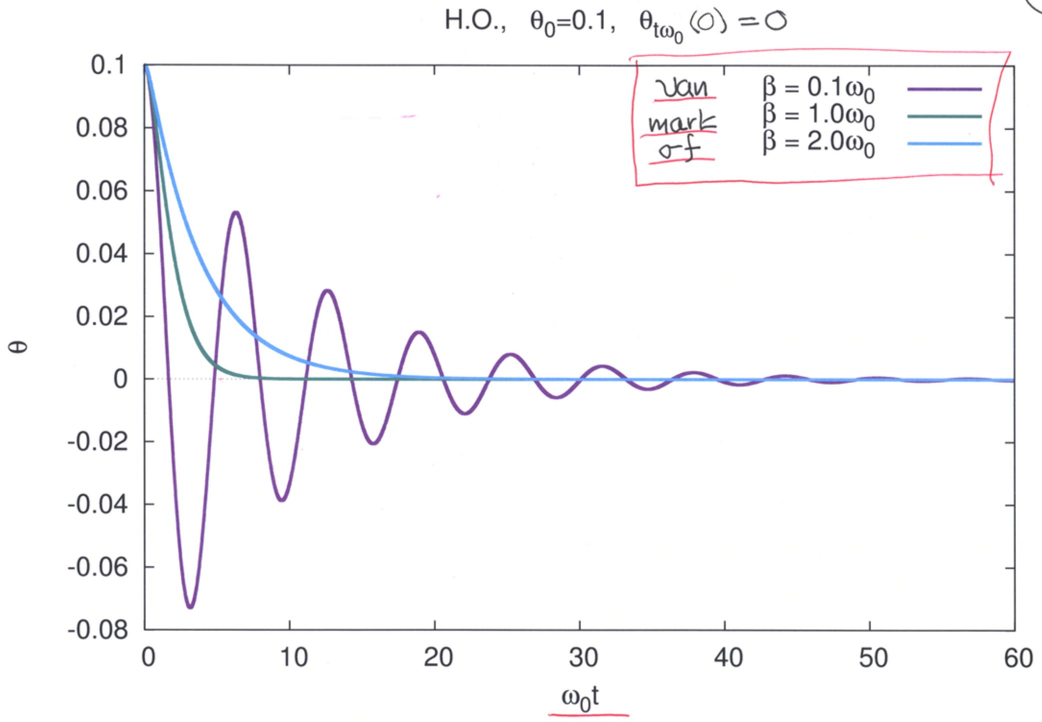
og lausnirnar eru ekki óháðar!

Rétt lausn þá er

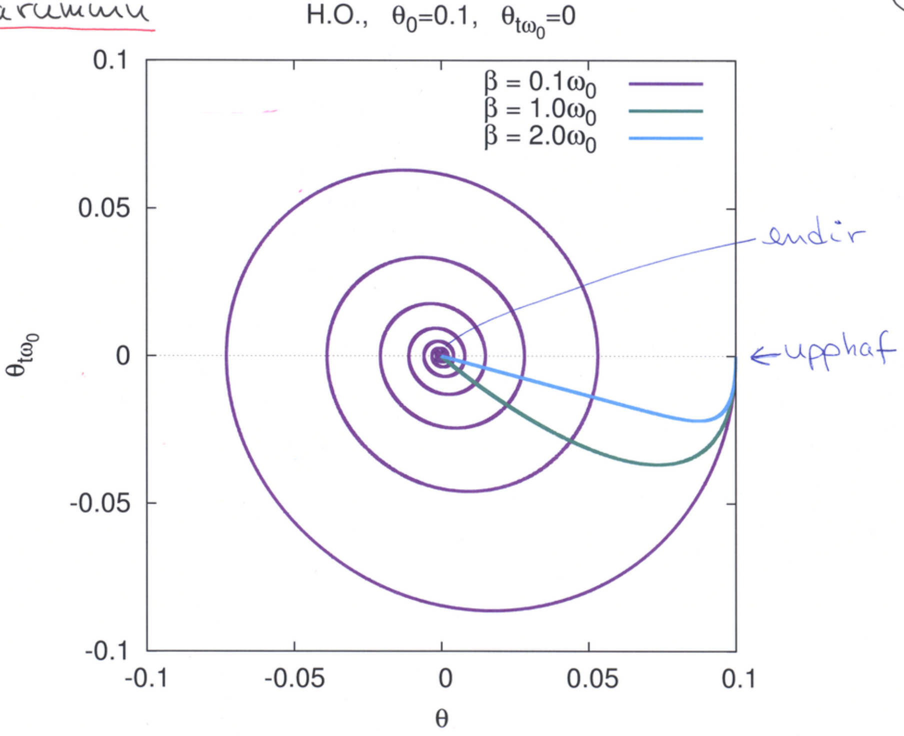
$$\Theta(t) = \left\{A + Bt\right\} e^{-\beta t}$$

Hún kallast markdeyfð sveifla

Skoðum tölulegar lausnir áður en við komum til baka að þessum lausnum


H.O., θ_0=0.1, $\theta_{t\omega_0}(0) = 0$
van
mark
of
$\beta = 0.1\omega_0$
$\beta = 1.0\omega_0$
$\beta = 2.0\omega_0$
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
θ
0
10
20
30
40
50
60
$\omega_0 t$

I fasarúminu

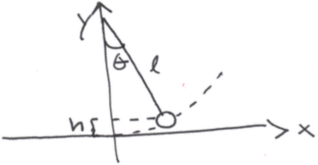


Heildarorka

$$E = T + V$$

Hreyfiorka + stöðuorka

$$E = \frac{1}{2} m v^2 + mgh$$



$$h = l - l\cos\theta$$

$$= l(1 - \cos\theta)$$

$$= l\left\{1 - \sqrt{1 - \sin^2\theta}\right\}$$

$$\simeq l\left\{1 - 1 + \frac{1}{2}\sin^2\theta + \cdots\right\}$$

$$\rightarrow h \simeq \frac{l}{2}\sin^2\theta \simeq \frac{l\theta^2}{2}$$

$$v = l\dot{\theta}$$

því fæst

$$E \simeq \frac{1}{2} m (l\dot{\theta})^2 + \frac{mgl}{2}\theta^2$$

$$= \frac{1}{2} m (l\dot{\theta})^2 + \frac{1}{2} m \frac{g}{l} (l\theta)^2$$

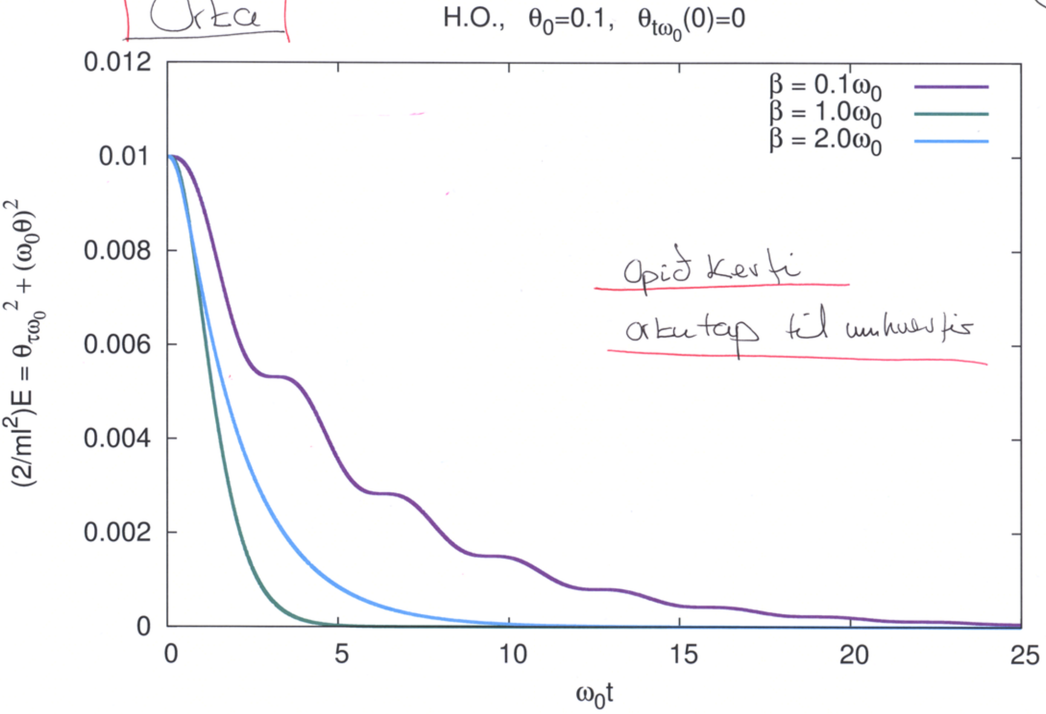
$$= \frac{1}{2} m \left\{(l\dot{\theta})^2 + \omega_0^2 (l\theta)^2\right\}$$

Hér erum við komin nærri, falli Hamiltons, sem við ræðum síðar.

$$\boxed{\frac{2}{ml^2} E = \dot{\theta}^2 + \omega_0^2 \theta^2}$$

Deyfingin kemur ekki beint fyrir hér

Orta


H.O., θ_0=0.1, $\theta_{t\omega_0}(0)$=0
$\beta = 0.1\omega_0$
$\beta = 1.0\omega_0$
$\beta = 2.0\omega_0$
$(2/ml^2)E = \theta_{\tau\omega_0}{}^2 + (\omega_0\theta)^2$
$\omega_0 t$
0
0.002
0.004
0.006
0.008
0.01
0.012
0
5
10
15
20
25
Opið kerfi
orkutap til umhverfis

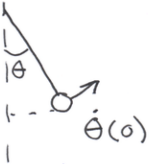
Ofdeyfð „sveifla"

$$\theta(t) = e^{-\beta t}\left[A_1 e^{\alpha t} + A_2 e^{-\alpha t}\right]$$

Báðir liðirnir eru í lausn, það skiptir máli hvernig $\dot{\theta}(0)$ er

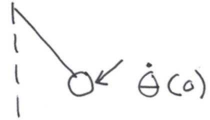
Þrjú tilfelli

1. $\dot{\theta}(0) > 0$



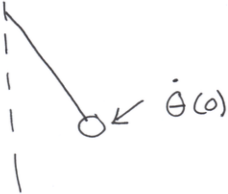
hámark fyrst svo fellur sveifillinn að núlli

2. $\dot{\theta}(0) < 0$

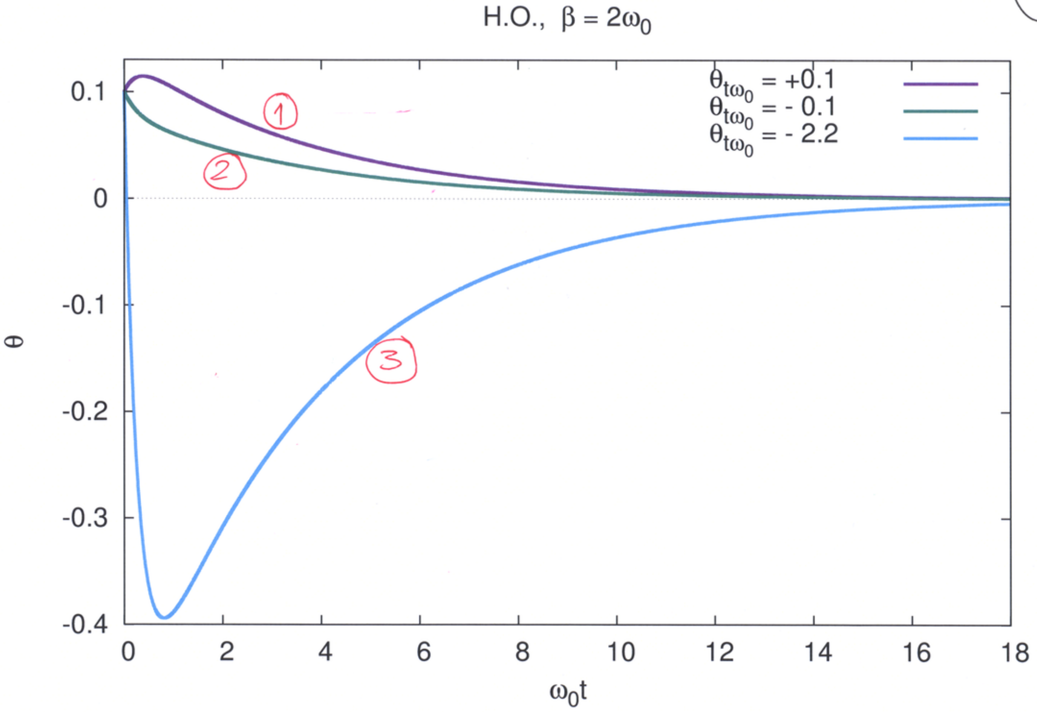


en samt nógu lítill hraði svo hann fellur beint að núlli

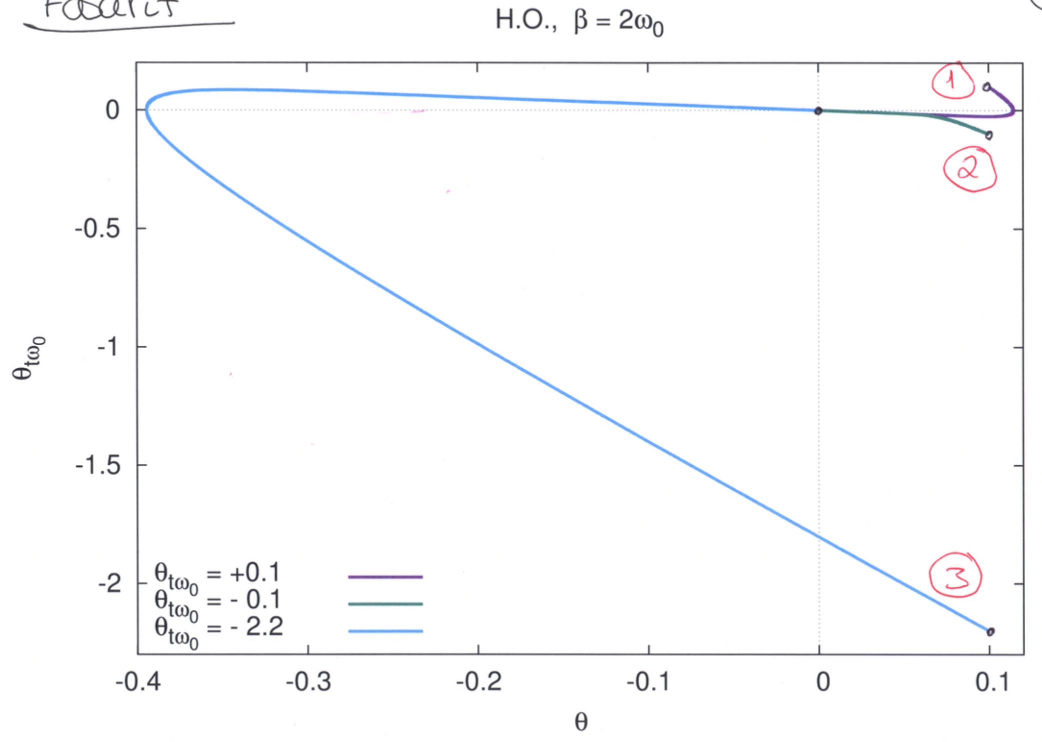
3. $\dot{\theta}(0) < -(\beta + \alpha)\theta(0)$



nógur hraði svo sveifillinn sveiflist einu sinni


H.O., β = 2ω0
θtω0 = +0.1
θtω0 = - 0.1
θtω0 = - 2.2
1
2
3
0.1
0
-0.1
-0.2
-0.3
-0.4
θ
0
2
4
6
8
10
12
14
16
18
ω0t

Fosarit



H.O., β = 2ω0
0
-0.5
-1
-1.5
-2
θtω0
-0.4
-0.3
-0.2
-0.1
0
0.1
θ
θtω0 = +0.1
θtω0 = - 0.1
θtω0 = - 2.2
1
2
3